Inflation, Default Risk and Nominal Debt

Carlo Galli
University College London

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Motivation

- Recent switch of many EM sovereigns to local-currency borrowing

- New issue arises
  - Strategic inflation as a way to alleviate debt burden
  - In addition to outright default

- Strategic inflation with nominal debt
  - Ex-post insurance benefits
  - Ex-ante time-consistency costs

- Joint behaviour of inflation and default spreads
  - Key for welfare implications of nominal debt
  - Linked to fiscal-monetary policy interaction in EM
Empirical Observations

- Asset price derivatives contain information on both risks, separately

- Common “printing press” argument does not hold
  - Default & inflation risks co-exist

- Default risk co-moves
  - With expected inflation
  - With realised inflation

...and this holds
  - Across countries, in long run
  - Within country, at short run frequencies
Theoretical Implications

Use facts to discipline quantitative sovereign default model

- Default as a binary choice
- Money (and inflation) as a continuous instrument
  1. dilutes real value of debt
  2. generates seignorage revenues

**Dilution motive** alone is counterfactual

- Inflation and default are substitutes
- Low incentive to inflate in bad times

**Revenue motive** reconciles model with data

- Seignorage flexible source of funding in bad times
- Inflation & default risks co-move
Takeaways

Default/inflation spreads drive government bond prices
- W/out commitment, determine costs of time-inconsistency
- Typically default spreads ↑ in bad times
- If inflation spreads co-move ⇒ debt policy even more constrained

Framework can be used to study
1. Welfare properties of LC debt issuance
2. Optimal fiscal-monetary setup (central bank commitment vs flexibility)

Role of expectations: low credibility → LC debt issuance costly outside of crisis

Potential implications
- Monetary-fiscal framework crucial for LC debt issuance
- Trade-off insurance vs. extra time inconsistency source
Related Literature

Time-consistent policy with nominal debt & default

Government debt currency denomination and “original sin”

Time-consistent policy with default & nominal rigidities
- Na et al. (2018), Bianchi et al. (2019), Arellano et al. (2019)

Currency and balance of payment crises
- Krugman (1979), Obstfeld (1986), Burnside et al. (2001)
Empirical Facts
Data Description

- Period: Jan 2004 - Feb 2019, quarterly
- Countries: Brazil, Colombia, Indonesia, Mexico, Malaysia, Poland, Russia, Thailand, Turkey, South Africa
  - all with freely/managed floating exchange rates (Ilzetzki et al., 2019)

![Average Share of Debt in Local Currency](chart)
Asset Price Data: Default Risk

Instrument: 5y Credit Default Swaps (CDSs)
- USD denominated, no currency risk
- Insure against default losses on international law debt
- Correlated with foreign-currency bond spreads
- Back out implied, risk-neutral default probability
Asset Price Data: Inflation Risk

Proxy with currency risk

Instrument: 5y Cross-Currency Swaps (XCSs)
- No credit risk, fully collateralised OTC derivatives
- Long-term analogue of implied yields in exchange rate forwards

\[ i - i^* = \frac{F_{wd}}{Spot} \]

- Interpret \( i - i^* \approx E_{\pi} - E_{\pi^*} \)
Fact 1: Long-Run, Across Countries

Cross-country averages for the period 2004q1-2018q4

![Graph showing the relationship between average annual default probability and average 5-year XCS for different countries.](image)

- BR, CO, ID, MX, MY, PL, RU, TH, TR, ZA
- Implied Default Prob.
- Post GFC
- IRS
Fact 2: Asset Price Correlation, Within Country

Time-series correlation between 5y default risk (CDS) & 5y currency risk (XCS)

Panel: $\hat{DP}_{i,t} = 0.437 \times XCS_{i,t}$ (two-way FE, SE 0.096)
Fact 3: Macro Correlations, Within Country

Time series correlation between
- 5y default risk (CDS) & nominal exchange rate (FX) yoy changes
- 5y default risk (CDS) & consumer price index (CPI) yoy changes

![Graph showing correlations between CDS-CPI and CDS-FX for different countries.](image_url)
Document co-movement

- Among asset prices: default risk and currency risk
- With macro variables: default risk and inflation/exchange rate depreciation
- In short & long run
Model
Environment

Quantitative, sovereign default model with
- Nominal debt
- Money
- Endogenous government spending

Players
- Benevolent government
- Domestic households
- Foreign lenders
Households

- Preferences: utility from real money balances (from $t-1$) and public good $g_t$

\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t U^h \left( c_t, \frac{M_t}{P_t}, g_t \right) = \frac{c_t^{1-\gamma}}{1-\gamma} + \alpha_m \frac{(M_t/P_t)^{1-\eta}}{1-\eta} + \alpha_g \frac{g_t^{1-\zeta}}{1-\zeta} \]

- Receive exogenous, stochastic income $y_t \sim AR(1)$
- Consume, pay taxes, hold money, save in domestic (zero net supply) bonds

\[ c_t + \frac{M_{t+1}}{P_t} + \frac{1}{R_t} \frac{B_{t+1}^d}{P_t} = \frac{M_t}{P_t} + \frac{B_t^d}{P_t} + y_t(1-\tau_t) \]

- Euler equations for domestic bonds

\[ \frac{1}{R_t} = \mathbb{E}_t \beta_h \left[ \frac{U_{c,t+1}^h}{U_{c,t}^h} \frac{P_t}{P_{t+1}} \right] \]

- Money demand equation

\[ R_t - 1 = \mathbb{E}_t \left[ \frac{U_{m,t+1}^h}{U_{c,t+1}^h} \right] \]
Government

- Benevolent, maximises households’ utility, own discount factor $\beta$, MIU wedge

\[ U \left( c_t, \frac{M_t}{P_t}, g_t \right) = \frac{c_t^{1-\gamma}}{1-\gamma} + (\alpha_m + \alpha_N) \frac{(M_t/P_t)^{1-\eta}}{1-\eta} + \alpha_g \frac{g_t^{1-\zeta}}{1-\zeta} \]

- No commitment to default & monetary policy

- Borrows externally, issues money domestically, chooses spending
  - “Benchmark” model: can also choose taxes freely
  - “Reduced” model: taxes are fixed

- Default implies
  - Exclusion from debt markets: receive offer to repay $B_t(1-h)$ & re-enter w.p. $\theta$
  - Reduced output $y^d(y_t) \leq y_t$
Timing

1) Start period with $B_t, M_t, y_t$

2) Government default/repay decision

3) Government fiscal/monetary policy decisions
   - Repay
     - issue $B_{t+1}$ to lenders at price $q_t$, choose $g_t, \tau_t, M_{t+1}$
     \[
     \tau_t y_t + q_t \frac{B_{t+1}}{P_t} + \frac{M_{t+1}}{P_t} = \frac{M_t}{P_t} + \frac{B_t}{P_t} + g_t
     \]
   - Default
     - Choose $g_t, \tau_t, M_{t+1}$
     \[
     \tau_t y^d(y_t) + \frac{M_{t+1}}{P_t} = \frac{M_t}{P_t} + g_t.
     \]

4) Households consumption/saving decisions
Lenders

- Risk-neutral, perfectly competitive, deep pockets
- Opportunity cost of funds $R^*$
- Zero-profit price of a unit of **new** government debt

$$q_t = \frac{1}{R^*} \mathbb{E}_t \left[ \frac{1 - \delta_{t+1}}{1 + \pi_{R,t+1}} + \frac{\delta_{t+1} q_{D,t+1}}{1 + \pi_{D,t+1}} \right]$$

- Zero-profit price of a unit of **defaulted** government debt

$$q_{D,t} = \frac{1}{R^*} \mathbb{E}_t \left[ (1 - \theta) \frac{q_{D,n,t+1}}{1 + \pi_{D,t+1}^n} + \theta \delta_{t+1} \frac{(1 - h)q_{D,o,t+1}}{1 + \pi_{D,t+1}^o} + \theta(1 - \delta_{t+1}) \frac{1 - h}{1 + \pi_{R,t+1}} \right]$$

**Implied expected default and inflation:**

- Default probability $D_{P,t} = \mathbb{E}_t \delta_{t+1}$
- Expected inflation $X_{CS,t} = \mathbb{E}_t \left[ \delta_{t+1} \pi_{D,t+1} + (1 - \delta_{t+1}) \pi_{R,t+1} \right]$
Private Sector Equilibrium

Focus on time-consistent, Markov-perfect equilibrium

- Gov't internalises effect of policy on future policies, prices and hhs' allocations

Recursive formulation

- Denote current, future variables with \((x, x')\)
- Make problem stationary → normalise nominal variables: \(\tilde{X} = X / M\)
- Aggregate state variables \((y, \tilde{B})\)

Given \(S := (\tilde{B}, y; \delta, g, \tau, \mu, \tilde{B}')\), Private Sector Equilibrium (PSE) is

- Household consumption policy \(c(S)\)
- Prices \(R(S)\) and \(m(S)\)
- Market clearing: money balances \((\tilde{M}'^d = 1)\), domestic bonds \((\tilde{B}'^d = 0)\).
Government Recursive Problem

- Default choice

\[
V(\tilde{B}, y) = \max_{\delta \in \{0, 1\}} (1 - \delta) V^R(\tilde{B}, y) + \delta V^D(\tilde{B}, y)
\]

- Repayment value

\[
V^R(\tilde{B}, y) = \max_{g, \tau, \mu, \tilde{B}'} U(c(S), m(S), g) + \beta \mathbb{E}_{y' | y} V(\tilde{B}', y')
\]

\[
\text{s.t. } y + q(S)\tilde{B}'(1 + \mu)m(S) = \underbrace{\tilde{B}m(S) + c(S) + g}_{qB'/P + B/P}
\]

- Default value

\[
V^D(\tilde{B}, y) = \max_{g, \mu} U(c(S), m(S), g) + \beta \mathbb{E} \left[ \theta V \left( \frac{\tilde{B}(1 - h)}{1 + \mu}, y' \right) + (1 - \theta) V^D \left( \frac{\tilde{B}}{1 + \mu}, y' \right) \right]
\]

\[
\text{s.t. } y^D(y) = c(S) + g
\]
Equilibrium

Definition (Markov-Perfect Equilibrium)

Given the aggregate state \( \{ \tilde{B}, y \} \), a recursive equilibrium consists of
- Government value functions \( V(\tilde{B}, y), V^R(\tilde{B}, y), V^D(\tilde{B}, y) \),
- Associated policy functions \( \delta(\tilde{B}, y), g(\tilde{B}, y), \tau(\tilde{B}, y), \mu(\tilde{B}, y) \) and \( \tilde{B}'(\tilde{B}, y) \)
- Private sector equilibrium \( \mathcal{P} \)

such that:

1. Value and policy functions solve the government problem, given \( \mathcal{P} \) and debt price functions \( q, q_D \)
2. \( \mathcal{P} \) is the PSE associated with government value and policy functions
Optimality: Repayment

Can summarise policy with \((c, \tilde{B}')\)

- back out \((g, \tau, \mu, m)\) from (RC) and PSE conditions

Inflation

- Benefit: ↓ real value of debt due \((\tilde{B}m)\) + ↑ tax revenues to finance \(g\)
- Cost: ↓ utility \((U_m)\)

Two first-order conditions:

Private-public consumption

\[
\frac{U_g - U_c}{\text{MC redistribution}} = m(c) \left( U_m - U_g \tilde{B} \right) \]

Euler equation

\[
\frac{U_g dr(\tilde{B}')}{\text{MR debt issuance}} + m(\tilde{B}') \left( U_m - U_g \tilde{B} \right) = \beta E U'_g m' \]

Backup MIU wedge

Carlo Galli (UCL)
Optimality: Default

Can summarise policy with $\mu$

- back out $(c, \tau, g, m)$ from (RC) and PSE conditions

Inflation

- Benefit: ↓ real debt due at re-entry + ↑ tax revenues to finance $g$
- Cost: ↓ utility ($U_m$)

First-order condition for $\mu$

\[
\frac{\partial}{\partial \mu} \beta \mathbb{E} \left[ (1 - \theta) V^D \left( y', \frac{\tilde{B}}{1 + \mu} \right) + \theta V \left( y', \frac{\tilde{B}(1 - h)}{1 + \mu} \right) \right] - c(\mu)(U_g - U_c) = -m(\mu) U_m
\]

↓ future debt burden

- MB redistribution
- MC ↓ real balances
Computation with Taste Shocks 1/2

Government recursive problem

1. Default choice

\[
V(\tilde{B}, y, \{\epsilon_R, \epsilon_D\}) = \max_{\delta \in \{0,1\}} \left\{(1 - \delta)[V^R(\tilde{B}, y) + \rho \delta \epsilon_R] + \delta[V^D(\tilde{B}, y) + \rho \delta \epsilon_D]\right\}
\]

2. Repayment value

\[
V^R(\tilde{B}, y, \{\epsilon_{\tilde{B}'}\}) = \max_{\tilde{B}'} \left\{W^R(\tilde{B}, y; \tilde{B}') + \rho_{\tilde{B}', \epsilon_{\tilde{B}'}}, \epsilon_{\tilde{B}'}\right\}
\]

where

\[
W^R(\tilde{B}, y; \tilde{B}') = U(c(\tilde{B}'), m(\tilde{B}'), g(\tilde{B}')) + \beta \mathbb{E}_{y'|y} V(\tilde{B}', y')
\]

3. Default value

\[
V^D(\tilde{B}, y, \{\epsilon_\mu\}) = \max_{\mu} \left\{W^D(\tilde{B}, y; \mu) + \rho_\mu \epsilon_\mu\right\}
\]

where

\[
W^D(\tilde{B}, y; \mu) = U(c(\mu), m(\mu), g(\mu)) + \beta \mathbb{E} \left[\theta V \left(\frac{\tilde{B}(1-h)}{1+\mu}, y'\right) + (1 - \theta) V^D \left(\frac{\tilde{B}}{1+\mu}, y'\right)\right]
\]
Computation with Taste Shocks 2/2

- \( \{\epsilon_R, \epsilon_D, \epsilon_{\tilde{B}^i}, \epsilon_{\mu}\} \sim \text{iid Gumbel}(-\bar{\mu}, 1) \)
- Choice probabilities:
  \[
  \mathbb{P}(x|\tilde{B}, y) = \frac{\exp \left[ W^i(\tilde{B}, y, x)/\rho_x \right]}{\sum_x \exp \left[ W^i(\tilde{B}, y, x)/\rho_x \right]}
  \]
- Expected values:
  \[
  V^i(\tilde{B}, y) = \rho_x \log \left\{ \sum_x \exp \left[ W^i(\tilde{B}, y, x)/\rho_x \right] \right\}
  \]

Magnitudes
- Consider choice \( x'' \) such that \( \log \frac{W^i(\tilde{B}, y; x'')}{\max_x W^i(\tilde{B}, y; x)} = -0.05\% \)
- \( \rho_{\tilde{B}^i} = 1e - 3 \)
- \( \rho_{\mu} = 5e - 3 \)
- \( \rho_{R,D} = 5e - 3 \)
Quantitative Evaluation

- Recall preferences
  \[ U(c, m, g) = \frac{c^{1-\gamma}}{1-\gamma} + (\alpha_m + \alpha_\nu) \frac{m^{1-\eta}}{1-\eta} + \alpha_g \frac{g^{1-\zeta}}{1-\zeta} \]

- Output process
  \[ \log(y_t) = \rho \log(y_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2_\epsilon) \]

- Default costs
  \[ y^d(y) = y - \max\{0, d_0 y + d_1 y^2\} \]

- External parameters:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private good utility curvature</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Money in utility curvature</td>
<td>$\eta$</td>
<td>3</td>
</tr>
<tr>
<td>International risk-free rate</td>
<td>$R^* - 1$</td>
<td>0.00598</td>
</tr>
<tr>
<td>Log-output autocorrelation</td>
<td>$\rho$</td>
<td>0.9293</td>
</tr>
<tr>
<td>Log-output innovation st. dev.</td>
<td>$\sigma_\epsilon$</td>
<td>0.0115</td>
</tr>
<tr>
<td>Re-entry probability</td>
<td>$\theta$</td>
<td>0.282</td>
</tr>
<tr>
<td>Recovery upon default</td>
<td>$1 - h$</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Benchmark Model

Assume lump-sum taxation available to the government
- Policy not constrained by Private Sector Equilibrium
- Govt can use $\tau$ to finance $g$
- Inflation not distorting $U_c - U_g$ margin, no wedges
- Public good utility curvature equal to private ($\zeta = 2$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Govt discount factor $\beta$</td>
<td>0.83</td>
<td>Debt service/GDP</td>
<td>0.058</td>
<td>0.099</td>
</tr>
<tr>
<td>Household discount factor $\beta_h$</td>
<td>0.99</td>
<td>Risk-free rate</td>
<td>0.073</td>
<td>0.064</td>
</tr>
<tr>
<td>MIU constant $\alpha_m$</td>
<td>2.7e-5</td>
<td>Monetary base/GDP</td>
<td>0.098</td>
<td>0.112</td>
</tr>
<tr>
<td>MIU constant (govt) $\alpha_\nu$</td>
<td>1.5e-3</td>
<td>CPI Inflation</td>
<td>0.049</td>
<td>0.038</td>
</tr>
<tr>
<td>Public good utility constant $\alpha_g$</td>
<td>0.07</td>
<td>$c/g$ ratio</td>
<td>3.67</td>
<td>3.66</td>
</tr>
<tr>
<td>Default cost parameter $d_0$</td>
<td>-0.3</td>
<td>Default prob. (mean)</td>
<td>0.045</td>
<td>0.029</td>
</tr>
<tr>
<td>Default cost parameter $d_1$</td>
<td>0.325</td>
<td>Default prob. (st. dev.)</td>
<td>0.020</td>
<td>0.052</td>
</tr>
</tbody>
</table>
Benchmark Model: Equilibrium Policy and Prices

Non-targeted moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(DP_t, XCS_t)$</td>
<td>-0.25</td>
<td>0.46</td>
</tr>
<tr>
<td>$\rho(y_t, XCS_t)$</td>
<td>0.43</td>
<td>0.02</td>
</tr>
<tr>
<td>$\rho(y_t, DP_t)$</td>
<td>-0.55</td>
<td>-0.2</td>
</tr>
<tr>
<td>$\rho(DP_t, \pi_t)$</td>
<td>0.02</td>
<td>0.31</td>
</tr>
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Real Debt Issuance $B'/P$

Exp. Default (solid), Exp. Inflation (dashed)
Reduced Model

Assume taxation is exogenous

- Fiscal capacity in EM typically low, hard to adjust
- Seignorage as a flexible source of funding
- Inflation tax distorts $U_c - U_g$ margin, wedges
- Public good utility curvature larger than private ($\zeta = 5$)

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<tr>
<td>MIU constant (govt)</td>
</tr>
<tr>
<td>Public good utility constant</td>
</tr>
<tr>
<td>Default cost parameter</td>
</tr>
<tr>
<td>Default cost parameter</td>
</tr>
<tr>
<td>Tax rate</td>
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Equilibrium Policy and Prices

Non-targeted moments

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Real Debt Issuance $B'/P$

Money Growth $\mu$

Exp. Default (solid), Exp. Inflation (dashed)
Takeaways

Counter-cyclical inflation

- Consistent with evidence in emerging market economies
- In bad times, strong motive to finance $g$ with inflation tax
  - not there with lump-sum taxation
- Matches co-movement btw default risk - inflation risk - realised inflation

Co-movement of inflation & default spreads

- Exacerbates time inconsistency $\rightarrow$ debt is costly when most needed
- Trade-off: insurance benefit vs. time-consistency costs relevant
  - Debt denomination
  - Central bank independence vs. fiscal flexibility
Conclusion

- Default risk co-moves with inflation risk, realised inflation and exchange rates
- Theory of monetary financing to match the data, debt dilution alone not enough
- Implications for debt currency denomination and fiscal-monetary interactions in economies with default risk
Appendix
Fact 1: Long-Run, Cross-Country

Cross-country averages for the period 2010q1-2018q4

[Two graphs showing correlations between Avg Annual Default Prob. (%) and Avg 5y XCS (%), and Avg YoY CPI (%) and Avg Annual Default Prob. (%)]
Fact 1: Long-Run, Cross-Country

Cross-country averages for the period 2004q1-2018q4

![Graphs showing the relationship between average annual default probability and average 5-year IRS, and average annual default probability and average YoY CPI across different countries.]
Fact 2: More Time-Series Correlation
## Data: Local-Currency Debt Focus

<table>
<thead>
<tr>
<th></th>
<th>Total Debt (% of GDP)</th>
<th>Foreign-Currency Debt (% of Total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>66.4</td>
<td>5.5</td>
</tr>
<tr>
<td>Colombia</td>
<td>39.2</td>
<td>28.6</td>
</tr>
<tr>
<td>Indonesia</td>
<td>33.2</td>
<td>41.0</td>
</tr>
<tr>
<td>Mexico</td>
<td>33.8</td>
<td>27.4</td>
</tr>
<tr>
<td>Malaysia</td>
<td>48.1</td>
<td>6.6</td>
</tr>
<tr>
<td>Poland</td>
<td>50.2</td>
<td>25.5</td>
</tr>
<tr>
<td>Russia</td>
<td>13.9</td>
<td>30.4</td>
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<tr>
<td>Thailand</td>
<td>27.3</td>
<td>2.3</td>
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<tr>
<td>Turkey</td>
<td>38.4</td>
<td>34.2</td>
</tr>
<tr>
<td>South Africa</td>
<td>38.7</td>
<td>11.4</td>
</tr>
</tbody>
</table>


- LC defaults as frequent as FC defaults
  - (post’97: 40 events, 35% FC, 25% LC, 32% both)
  - (post’75: 63 events, 43% FC, 33% LC, 24% both)

- Same credit ratings on LC & FC debt

(source: Moody’s sector in-depth (02/04/2019))
### Descriptive Statistics (2004m1-2019m2)

<table>
<thead>
<tr>
<th></th>
<th>CPI yoy</th>
<th>FX yoy</th>
<th>IRS 5y</th>
<th>CDS 5y</th>
<th>Debt/GDP (%)</th>
<th>FC Debt Share (%)</th>
<th>Ext Debt Share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR</td>
<td>5.7 (1.8)</td>
<td>3.1 (19.3)</td>
<td>9.2 (1.9)</td>
<td>2.2 (1.3)</td>
<td>66.4</td>
<td>5.5</td>
<td>13.3</td>
</tr>
<tr>
<td>CO</td>
<td>4.4 (1.7)</td>
<td>1.3 (15.1)</td>
<td>6.5 (1.8)</td>
<td>1.8 (1)</td>
<td>39.2</td>
<td>28.6</td>
<td>37.7</td>
</tr>
<tr>
<td>ID</td>
<td>6.4 (3.4)</td>
<td>3.9 (9.8)</td>
<td>8.4 (2.3)</td>
<td>2.0 (1.2)</td>
<td>33.2</td>
<td>41.0</td>
<td>55.1</td>
</tr>
<tr>
<td>MX</td>
<td>4.2 (1)</td>
<td>4.4 (11)</td>
<td>7.1 (1.6)</td>
<td>1.2 (0.6)</td>
<td>33.8</td>
<td>27.4</td>
<td>30.6</td>
</tr>
<tr>
<td>MY</td>
<td>2.5 (1.6)</td>
<td>0.8 (8.2)</td>
<td>3.8 (0.4)</td>
<td>1.1 (0.4)</td>
<td>48.1</td>
<td>6.6</td>
<td>27.1</td>
</tr>
<tr>
<td>PL</td>
<td>2.0 (1.7)</td>
<td>0.7 (15.4)</td>
<td>4.2 (1.6)</td>
<td>1.1 (0.6)</td>
<td>50.2</td>
<td>25.5</td>
<td>44.7</td>
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<tr>
<td>RU</td>
<td>8.8 (3.7)</td>
<td>6.6 (20.3)</td>
<td>8.0 (3.2)</td>
<td>2.2 (1.3)</td>
<td>13.9</td>
<td>30.4</td>
<td>29.2</td>
</tr>
<tr>
<td>TH</td>
<td>2.3 (2.2)</td>
<td>-1.5 (6)</td>
<td>3.0 (1)</td>
<td>1.1 (0.5)</td>
<td>27.3</td>
<td>2.3</td>
<td>11.0</td>
</tr>
<tr>
<td>TR</td>
<td>9.1 (3)</td>
<td>9.6 (16.5)</td>
<td>11.3 (3.8)</td>
<td>2.4 (0.9)</td>
<td>38.4</td>
<td>34.2</td>
<td>30.2</td>
</tr>
<tr>
<td>ZA</td>
<td>5.5 (2.3)</td>
<td>5.1 (14.8)</td>
<td>8.0 (1.1)</td>
<td>1.6 (0.8)</td>
<td>38.7</td>
<td>11.4</td>
<td>27.7</td>
</tr>
</tbody>
</table>
Table: Time series regression and variance-covariance decomposition of 5y LC bond yields monthly changes, for the period Jan 2004 - Feb 2019. HAC robust standard errors used in all regressions, significance levels indicated by *** (p<0.01), ** (p<0.05), * (p<0.1).

<table>
<thead>
<tr>
<th>Country</th>
<th>$R^2$</th>
<th>IRS %</th>
<th>CDS %</th>
<th>Covariance %</th>
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<tr>
<td>BR</td>
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<td>CO</td>
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<td>3</td>
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<tr>
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<td>85</td>
<td>7</td>
<td>8</td>
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<tr>
<td>TH</td>
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<tr>
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<td>59</td>
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</tr>
<tr>
<td>ZA</td>
<td>0.91</td>
<td>93</td>
<td>1</td>
<td>6</td>
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</tbody>
</table>
Asset Price Details: Default Risk

CDSs:
- Pay a periodic premium (spread) in exchange for default “insurance”
- Credit event: change in interest, principal, postponement of interest/principal, change in currency or seniority
- Upon credit event: protection buyer has option to deliver to seller an acceptable bond in a permitted currency
- Deliverable currencies typically include USD, EUR, YEN; GBP, CHF, CAD, AUD

Moody’s sector in-depth (2019)
- LC defaults as frequent as FC defaults
  - post’97: 40 events, 35% FC, 25% LC, 32% both
  - post’75: 63 events, 43% FC, 33% LC, 24% both
- Same credit ratings on LC & FC debt
CDS-Implied Default Probabilities

- Survival prob. with default intensity $\lambda(t)$: $S(t) = Pe^{-\int_0^t \lambda(u)du}$
- Premium leg: $PV$ of all premium payments
  \[
P_{\text{Prem}} = \mathbb{E} \int_0^T DF(t) U_{\text{par}} \mathbb{1}[T_1 > t] = U_{\text{par}} \int_0^T DF(t) S(t) dt.
\]
- Protection leg: $PV$ of $LGD$, at random time $T_1 | T_1 < T_{\text{expiry}}$
  \[
P_{\text{Prot}} = \mathbb{E} \{ DF(T_1) \times LGD \times \mathbb{1}[T_1 \leq T] \} = LGD \int_0^T DF(t) S(t) \lambda(t) dt.
\]
- Par spread is given by
  \[
  U_{\text{par}} = \frac{LGD \int_0^T DF(t) S(t) \lambda(t) dt}{\int_0^T DF(t) S(t) dt}.
\]
- Assume: constant hazard rate ($\lambda(t) = \lambda$): $\lambda = \frac{U_{\text{par}}}{LGD}$
- Default probability thus given by
  \[
  \text{DefProb}_t = 1 - S(t) = 1 - e^{-\lambda t} = 1 - e^{-\frac{U_{\text{par}}}{LGD} t}.
  \]
Asset Price Details: Inflation Risk

IRRs:
- pay/receive periodic fixed rate for local LIBOR (≈ key CB rate)
- constant maturity, fully collateralised OTC derivatives

Fixed-for-Fixed Cross-Currency Swaps (Du-Schreger, 2016):
- when Non-Deliverable Cross-Currency Swaps are available
  - NDS fixed-for-floating: LC fixed ↔ USD LIBOR
  - Plain USD IRS: USD LIBOR ↔ USD fixed

- when Cross-Currency Swap Basis is available
  - Plain LC IRS: LC fixed ↔ LC LIBOR
  - XC Basis: LC LIBOR ↔ USD LIBOR
  - Plain USD IRS: USD LIBOR ↔ USD fixed
Repayment Problem

- Plugging in \( q(y, \tilde{B}') \) and \( m := 1/\tilde{P} \) simplifies the resource constraint to

\[
y + \frac{\mathbb{E}^q(\tilde{B}')\tilde{B}'}{R^*} - \tilde{B}m = c + g
\]

where

\[
\mathbb{E}^q(\tilde{B}') = \mathbb{E} \left[ (1 - \delta')m'_R + \delta' q_D(y', \tilde{B}')m'_D \right]
\]

- Households’ real money demand (omitting \( y \))

\[
(1 + \mu)m = \mathcal{M}^d(c, \tilde{B}') := \frac{\beta h}{U_c} \mathbb{E} \left[ (U'_c + U'_m)m' \right]
\]

- Plug \( \mathcal{M}^d \) into hh BC yields \( m(c, \tilde{B}') \)

\[
c + \mathcal{M}^d(c, \tilde{B}') = m + y(1 - \tau)
\]

- Get \( \mu(c, \tilde{B}') \) from either money demand or hh BC
Households’ real money demand

\[(1 + \mu)m = \beta_h \frac{\mu}{u'(c)} \mathbb{E} [(U'_c + U'_m)m']\]

Combining it with the hh BC, get

\[c(\mu) = \left\{ c : u'(c)[y_D(1 - \tau) - c] = \beta_h \frac{\mu}{1 + \mu} \mathbb{E} [(U'_c + U'_m)m'] \right\}\]

Get \(m(\mu)\) from either money demand or hh BC
Controlling for a Global Factor

![Defprob5y on XCS 5y](image)

- MY, TR, MX, CO, TH, BR, RU, PL, ZA, ID
- CI 95%
- CI 95%
- Beta
- Panel
Private Sector Equilibrium

Definition (Private Sector Equilibrium (PSE))

Given $S := (\tilde{B}, y; \delta, g, \tau, \mu, \tilde{B}')$, a symmetric PSE consists of

- Household policies $c(S)$, $\tilde{M}'^d(S)$ and $\tilde{B}'^d(S)$,
- The risk-free rate $R(S)$ and the inverse of the price level $m(S)$

such that:

1. Policies solve the household problem;
2. Market clearing: money balances ($\tilde{M}'^d = 1$), domestic bonds ($\tilde{B}'^d = 0$).
Lenders conditions, recursive formulation

- Inflation $1 + \pi' := \frac{\tilde{p}'(1+\mu)}{\tilde{p}}$

- Price of new debt, upon repayment (omitting $y$)

$$q(S) = \frac{1}{R^*} \frac{\tilde{p}_R(S)}{1 + \mu} \mathbb{E} \left[ \frac{1 - \mathbb{H}_D(y', \tilde{B}')}{\tilde{p}_R'(y', \tilde{B}')} + \mathbb{H}_D(y', \tilde{B}') \frac{q_D(y', \tilde{B}')}{{\tilde{p}_R}'(y', \tilde{B}')} \right]$$

- Price of defaulted debt

$$q_D(S) = \frac{1}{R^*} \frac{\tilde{p}_D(S)}{1 + \mu} \mathbb{E} \left\{ (1 - \theta) \frac{q_D'(y', \tilde{B}^n)}{\tilde{p}_D'(y', \tilde{B}^n)} ight. $$

$$+ \theta (1 - h) \left[ \mathbb{H}_D(y', \tilde{B}^o) \frac{q_D'(y', \tilde{B}^o)}{\tilde{p}_D'(y', \tilde{B}^o)} + \frac{1 - \mathbb{H}_D(y', \tilde{B}^o)}{\tilde{p}_R'(y', \tilde{B}^o)} \right] \right\}$$

where $\tilde{B}^n := \tilde{B} / (1 + \mu)$, $\tilde{B}^o := (1 - h)\tilde{B} / (1 + \mu)$

- Default probability $DP(y, \tilde{B}') = \mathbb{E}_{y'|y} \mathbb{H}_D(y', \tilde{B}')$

- Expected inflation

$$XCS(S) = \frac{1 + \mu}{\tilde{p}(S)} \mathbb{E}_{y'|y} \left\{ \mathbb{H}_D(y', \tilde{B}') \tilde{p}_D'(y', \tilde{B}') + [1 - \mathbb{H}_D(y', \tilde{B}')]\tilde{p}_R'(y', \tilde{B}') \right\}$$
Money Demand Elasticity

Taking the money demand equation

\[ i_{d,t+1} = \mathbb{E}_t \frac{\alpha m (m_{t+1})^{-\eta}}{c_{t+1}} \]

and linearising

\[ \mathbb{E} \log(M_{t+1}/P_{t+1}) = \frac{\text{const}}{\eta} + \frac{\gamma}{\eta} \mathbb{E} \log c_{t+1} - \frac{1}{i_d \eta} i_{d,t+1} \]

which implies

- **Semi-elasticity** = \( \frac{1}{i_d \eta} \)
- **Elasticity** = \( \frac{1}{\eta} \)
Inflation Expectations Cyclicality

- Defined as
  \[
  \frac{\partial XCS(y, B')}{\partial y} = \frac{\partial}{\partial y} \int [\delta' \pi_D' + (1 - \delta') \pi_R'] f(y', y) dy'
  \]

→ to co-move with default risk, need counter-cyclical XCS

Decompose

\[
\begin{align*}
\frac{\partial \tilde{B}'}{\partial y} & \left[ \frac{\delta' \partial \pi_D'}{\partial \tilde{B}'} + \frac{(1 - \delta') \partial \pi_R'}{\partial \tilde{B}'} \right] dF(y'|y)^

(a) \frac{\partial \pi'}{\partial \tilde{B}'} \text{ effect: } > 0

P-mass↑

+ \int_{\hat{y}}^{y} \pi_D' \frac{\partial f(y'|y)}{\partial y} dy' + \int_{\hat{y}}^{y} \pi_R' \frac{\partial f(y'|y)}{\partial y} dy' + \int_{\hat{y}}^{y} \pi_R' \frac{\partial f(y'|y)}{\partial y} dy'

(b) \frac{\partial \pi'}{\partial y'} \text{ effect: } < 0?

P-mass↓

- \frac{\partial \tilde{B}'}{\partial y} \frac{\partial \hat{y}}{\partial \tilde{B}'} [\pi_R'(\tilde{B}', \hat{y}) - \pi_D'(\tilde{B}', \hat{y})] f(\hat{y}|y)

(c) cutoff effect

P-mass↑
MIU Wedge

The benchmark model FOC yield

\[ U_c = U_g \]
\[ U_m = U_g \tilde{B} \]

Household money demand

\[ R - 1 = \mathbb{E} \frac{U_{hh}^m}{U_{hh}'} \]

Combining the two equations

\[ R - 1 = \tilde{B}' \mathbb{E} \frac{U_{hh}^m}{U_{m}'} \]
Money Growth and Seignorage

- Recall HH money demand $R - 1 = E \alpha_m (M'/P')^{-\eta}/c'^{-\gamma}$
- An increase in $\mu$ or in $M'$
  - $\downarrow R, c \Rightarrow \downarrow$ real money demand
  - $\downarrow m, \uparrow$ seignorage $m\mu$

Changes in Money Growth $\mu$, at $y = 1, \hat{B} = 0.4$