The Paper

- CB balance sheet policies (QE & FX interventions)
  - Empirics: debated yet relevant effects on asset prices
  - Theory: policy is irrelevant in a frictionless world (Wallace (1981))

- Friction: bounded rationality (level-k thinking)

- Main results:
  1. Level-k thinking makes policy relevant, in various settings
  2. Generates forecast errors related to policy → consistent with data
Discussion Points

▶ Bounded rationality: what is level-k thinking?

▶ Asset pricing application → 2 questions to be asked
  1. Micro: how does it work, what do we learn?
  2. Macro: is the application appropriate?
Level-k Thinking in Beauty Contests

- Nash equilibrium implies
  - agents have a high degree of rationality
  - agents assume others have a high degree of rationality
- Many experimental results violate this
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p-Beauty Contest game

- $N$ players, each picks a number $s_i \in \{1, 2, ..., 100\}$
- closest to $p \times \frac{\sum_i s_i}{N}$ wins, with $p \in (0, 1)$
- iterated deletion of dominated strategies:
  1. even if all play 100, I should guess $p \times 100$
  2. if all play $p \times 100$, I should guess $p^2 \times 100$
  3. and so on... → Nash Eqm is 1
p=2/3 Beauty Contest Game

Nagel (1995)

- if people play at (uniform) random → 50 (level-0, non-strategic)
- if people best-respond to level-0 → 33 (level-1)
- if people best-respond to level-1 → 22 (level-2)
- and so on...
Asset Prices and Balance Sheet Policies

Infinite horizon $t = 1, 2, ...$

Markets

- risky asset, pays $r_{t+1}^x \sim N(r^x, \sigma^2)$ each period, fixed supply $\tilde{X}$
- risk-free asset in infinite supply with gross return $R$

Agents

- have CARA utility $U(c_{t+1}) = -e^{-\gamma c_{t+1}}$
- OLG, agents live 2 periods, born with $(w)$, consume only when old

\[
c_{t+1} - wR = \left( r_{t+1}^x + q_{t+1} - q_t R \right) x_{t+1} - T_{t+1}
\]

Government

- finances risky-asset purchases with risk-free debt $\rightarrow B_{t+1} = q_t X_{t+1}^G$
- transfers profits to old agents

\[
-T_{t+1} = \mathcal{R}_{t+1} X_{t+1}^G
\]
Asset Prices and Balance Sheet Policies

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Markets
- risky asset, pays $r_{t+1}^X \sim N(r^X, \sigma^2)$ each period, fixed supply $\bar{X}$
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- have CARA utility $U(c_{t+1}) = -e^{-\gamma c_{t+1}}$
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\[ c_{t+1} - wR = (r_{t+1}^X + q_{t+1} - q_t R)(x_{t+1} + X_{t+1}^G) \]

Government
- finances risky-asset purchases with risk-free debt $\rightarrow B_{t+1} = q_t X_{t+1}^G$
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\[ -T_{t+1} = R_{t+1} X_{t+1}^G \]
Rational Expectations Equilibrium

- CARA-Normal $\Rightarrow$ mean-variance maximization

\[ x_{t+1} = \frac{E_t(R_{t+1})}{\gamma \text{Var}(R_{t+1})} - X_{t+1} \quad \Rightarrow \quad q_{t}^{\text{REE}} = \left( q_{t+1}^{\text{REE}} + r^x - \gamma \sigma^2 \bar{X} \right) / R \]

- REE price is present expected value of risk-adjusted dividends,

\[ q_{t}^{\text{REE}} = \frac{r^x - \gamma \sigma^2 \bar{X}}{R - 1} \]

- $q_{t}^{\text{REE}}$ $\perp \{X_{t}^{G}\}_{t \geq 0}$: QE crowds out private investment
Temporary Equilibria, 1-period QE ($X_3 > 0$)

- Status-quo: no QE ($T_t = 0, X_t^G = 0 \ \forall t$)
- At $t = 0$, one-period QE announcement: $X_3^G > 0 \rightarrow T_3 = -R_3 X_3^G$
  - policy is known to all $k$-types

($k = 1$)

- agents’ beliefs = status-quo eqm distribution
- $\tilde{E}_{t}^{k=1}(q_{t+1}) = q_{t+1}^{REE}$, still $T_3 = 0$
- asset demand in $t = 2$

$$X_3^{k=1} = \frac{\tilde{E}_{t}^{k=1}(R_3)}{\gamma \text{Var}(R_3)}$$
Temporary Equilibria, 1-period QE ($X_3 > 0$)

- Status-quo: no QE ($T_t = 0, X_t^G = 0 \forall t$)
- At $t = 0$, one-period QE announcement: $X^G_3 > 0 \rightarrow T_3 = -\mathcal{R}_3 X^G_3$
  - policy is known to all $k$-types

$(k = 1)$

- agents’ beliefs = status-quo eqm distribution
- $\tilde{E}^{k=1}(q_{t+1}) = q^{REE}_{t+1}$, still $T_3 = 0$
- asset demand in $t = 2$

$$x^{k=1}_3 = \frac{\tilde{E}^{k=1}(\mathcal{R}_3)}{\gamma \text{Var}(\mathcal{R}_3)}$$

- $k = 1$ temporary eqm prices

$$q^{k=1}_{t<2} = q^{REE}$$

$$q^{k=1}_2 = q^{REE} + \frac{r^x - \gamma \sigma^2 (\bar{X} - X^G_3)}{R} = q^{REE} + \frac{\gamma \sigma^2 X^G_3}{R}$$
Temporary Equilibria, 1-period QE ($X_3 > 0$)

($k = 2$)

- agents beliefs = eqm distribution if everyone is ($k = 1$)

- $\tilde{E}_{1}^{k=2}(q_2) = q_{2}^{k=1}

- understand taxes are risky:

\[
T_3 = -\mathcal{R}_{3}^{k=1}X_{3}^{G}, \quad X_{3}^{k=2} = \frac{\tilde{E}_{t}^{k=1}(\mathcal{R}_3)}{\gamma \text{Var} \left( \mathcal{R}_3 \right)} - X_{3}^{G}
\]

- $k = 2$ temporary eqm prices

\[
q_{2}^{k=2} = q^{REE} \perp X_{3}^{G}
\]
Temporary Equilibria, 1-period QE ($X_3 > 0$)

($k = 2$)

- agents beliefs = eqm distribution if everyone is ($k = 1$)

- $\tilde{E}^{k=2}_1(q_2) = q^{k=1}_2$

- understand taxes are risky:

\[
T_3 = -R^{k=1}_3 X^G_3, \quad X^{k=2}_3 = \frac{\tilde{E}^{k=1}_1(R_3)}{\gamma \text{Var}(R_3)} - X^G_3
\]

- $k = 2$ temporary eqm prices

\[
q^{k=2}_2 = q^{\text{REE}} \perp X^G_3
\]

\[
q^{k=2}_1 = \frac{q^{k=1}_2 + r^x - \gamma \sigma^2 \bar{X}}{R} = q^{\text{REE}} + \frac{\gamma \sigma^2 X^G_3}{R^2}
\]
Temporary Equilibria, 1-period QE ($X_3 > 0$)

To simplify, let risk-adjusted expected dividend ($r^x - \gamma \sigma^2 \bar{X}$) = 0

$\Rightarrow t = 0$ effect of $X_t^G$ only for $k = t$ agents
Reflective Equilibrium, multi-period QE ($X_t = \delta^{t-1}$)
Comments

1. **k-type distribution assumed constant over time.** What if $k$-types are long-lived?
   - lower $k \rightarrow$ largest positions/risks
   - with exit, mass ↑ and QE effect weaker $\approx$ effects of learning in paper
Discussion

2. **k believes everyone else is k − 1**: strong “illusory superiority”
   - what if agents know the type distribution?

3. **k ≥ 2 get Cov(\(R_{t+1}, T_{t+1}\)) perfectly**
   - no within-period QE effects for \(k ≥ 2\)
   - ≠ Fahri and Werning (2016), Garcia-Schmidt and Woodford (2015)
   - **static beauty contest ≈ dynamic sequential trading**?

4. **Gov’t agencies large players in mortgage market for decades**
   - are gov’t balance sheet policies really novel for mkt participants?
   - Fieldhouse et al. (2018)
Bottomline

- Nice, clear, novel asset pricing application of level-k expectations

- Application hinges on restrictions *within* level-k thinking
  - results somewhat robust to learning and (some) rational agents

- Choice of bounded rationality/information friction
  - empirical justification from forecast errors seems right way to go