Asset Purchases and Default-Inflation Risks when Investors Learn from Prices

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Motivation

APs typically a monetary policy tool when at the ZLB
- reduce long term rates
- restore appropriate function of monetary policy transmission mechanism

APs as a *fiscal* tool, to prevent sovereign debt crises and support govt debt service
...or not?

*Lagarde: We are not here to close spreads, there are other tools and other actors to deal with these issues*

3:10 PM · Mar 12, 2020 · Twitter Web App

“The problem with QE is that it works in practice, but it does not work in theory”
Ben Bernanke, January 16th, 2014
Contribution

1. Can APs compress credit spreads? How? When is that useful?
2. How do APs affect the information contained in market prices?

We answer these questions with a model that features

- fiscal-monetary interactions (Sargent and Wallace, 1981)
- sovereign default (Eaton and Gersovitz, 1981)
- noisy financial markets (Hellwig, Mukherji and Tsyvinski, 2006)
What we find

Asset purchases

- expose the CB balance sheet (hence inflation) to default risk
- crowd out private investors
  - relevant if beliefs are heterogeneous
  - reduce nominal and real sovereign yields
  - affect the informational content of market prices, asymmetrically
- net welfare effect \(> 0\) under some conditions

⇒ information frictions as a rationale for why APs may work “in theory”

Implications

- degree of belief heterogeneity key for AP elasticity of the interest rate
Outline

1. Model setup
2. Homogeneous information
3. Heterogeneous information
Model: Government

Two periods, \( t = 1, 2 \)

First period \( t = 1 \)
- fund stochastic spending by issuing nominal + defaultable debt
  \[
  b = S \quad \text{where } S \sim U[0, 1]
  \]

Second period \( t = 2 \)
- raise taxes, can default (\( \delta \in \{0, 1\} \)) with haircut \( h \) and deadweight loss \( \theta \)
  \[
  b \frac{R(1 - \delta h)}{\Pi} = \tau S \quad \rightarrow \quad \underbrace{\frac{R(1 - \delta h)}{\Pi}}_{\psi(R, \Pi, \delta)} = \tau
  \]
- \( \delta \) decision minimises distortions from taxes (\( \zeta \)) & default (\( \theta \))
  \[
  \mathcal{L} := (1 - \delta)\zeta(\psi(R, \Pi, 0)) + \delta [\zeta(\psi(R, \Pi, 1)) + \theta]
  \]
  default iff \( \zeta(\psi(R, \Pi, 0)) > \zeta(\psi(R, \Pi, 1)) + \theta \)
  for today, default iff \( \theta < \hat{\theta} \)
Model: Households

Continuum of risk-neutral agents \( i \in [0, 1] \)

First period \( t = 1 \)
- receive information on APs, \( R \) and \( \theta \) (with noise)
- receive endowment \( e_1 \), save it in 3 assets
  \[ e_1 \geq b^i + m^i + s^i \]

Second period \( t = 2 \)
\[ c^i = b^i \frac{R(1 - \delta h)}{\Pi} + \frac{m^i}{\Pi} + \rho s^i - \tau S - \mathcal{L} \]
- pay taxes, consume
- tax & default distortions \( \mathcal{L} \) create deadweight losses
First period $t = 1$: issue money, save via storage (real + risk-free) or bonds

$$s^{cb} + b^{cb} = m \quad \rightarrow \quad s^{cb} \frac{m}{m} = 1 - \alpha$$

Second period $t = 2$: reimburse money with returns from saving

$$\rho s^{cb} + \frac{b^{cb} R (1 - \delta h)}{\Pi} = \frac{m}{\Pi} \quad \rightarrow \quad (1 - \alpha) \rho + \alpha \frac{R (1 - \delta h)}{\Pi} = \frac{1}{\Pi}$$

Let share of money invested in bonds be $\alpha := \frac{b}{m}$

- return of money as $\alpha$-weighted average
- $\alpha \rightarrow$ degree of fiscal dominance
Model: Central Bank /2

Solving for the real return on money

\[ \frac{1}{\Pi} \left[ 1 - \alpha R(1 - \delta h) \right] = \rho(1 - \alpha) \]

- net return
- nominal liabilities
- real assets

Plug into real bond returns

\[ \psi(R, \alpha, \delta) = \frac{R(1 - \delta h)}{\Pi(R, \delta, \alpha)} = \rho \frac{1 - \alpha}{R(1 - \delta h)} - \alpha \]

Note that \( R \in \left[ 1, \frac{1}{1-h} \right] \)

- Repayment: \( \delta = 0 \) and \( R > 1 \)
  - CB makes profits, \( \frac{1}{\Pi} > \rho \)
  - \( \uparrow \alpha \Rightarrow \downarrow \Pi \Rightarrow \uparrow \psi \)

- Default: \( \delta = 1 \) and \( R(1 - h) < 1 \)
  - CB makes losses, \( \frac{1}{\Pi} < \rho \)
  - \( \uparrow \alpha \Rightarrow \uparrow \Pi \Rightarrow \downarrow \psi \)
Market clearing

Bonds market clearing

\[ \int b^i di + b^{cb} = b \]

Goods market clearing

\[ c = \rho [e_1 - S] - L \]

Timing

1. First period
   1.1 asset purchases \( \alpha \) are unconditional, CB does not observe shocks
   1.2 shocks \((\theta, S)\) realise
   1.3 agents receive information and make portfolio decisions

2. Second period
   2.1 government observes shocks perfectly, takes default decision
   2.2 payoffs realise & agents consume
Roadmap

1. Perfect foresight
2. Uncertainty + homogeneous information
3. Uncertainty + heterogeneous information & learning from prices
1) Perfect foresight

Everyone knows $\delta = \mathbb{1}[\theta < \hat{\theta}]$

Equilibrium interest rate

$$R(1 - \delta h) = 1$$

Inflation is anchored

$$\frac{1}{\Pi} = \rho \perp \alpha$$

Real bond return = real money return = \(\rho\)

$$\Rightarrow \text{Asset purchases are irrelevant}$$
2) Uncertainty + homogeneous information

Agents and CB share same uncertainty: \( \text{Prob}(\delta = 0) = p \)

Equilibrium \( R \) solves no-arbitrage condition

\[
p \psi(R, \alpha, 0) + (1 - p) \psi(R, \alpha, 1) = 1
\]

Expected welfare loss

\[
p \zeta \left( \psi(R, \alpha, 0) \right) + (1 - p) \zeta \left( \psi(R, \alpha, 1) \right)
\]

Effect of asset purchases

- ↑ CB exposure to default risk
- increase the variance of inflation (↑ \( \mathbb{V}(\Pi) \)) and real bond returns (↑ \( \mathbb{V}(\psi) \))
- but \( \mathbb{E}(\psi) \leftrightarrow \) by no-arbitrage

⇒ With convex distortions, \( \alpha^* = 0 \)
3) Uncertainty + heterogeneous information & learning from prices

For simplicity, assume that the fundamental

\[ \theta = \begin{cases} 
\theta^H \text{ (repay)} & \text{w.p. } q \\
\theta^L \text{ (default)} & \text{w.p. } 1 - q 
\end{cases} \]

Agent \( i \) observes
- private signal \( x_i = \theta + \sigma_x \xi \) where \( \xi \sim N(0, 1) \)
- equilibrium price \( R \) (endogenous public signal)
\[ \Rightarrow \text{subjective repayment probability } p(x_i, R, \alpha) \]

Assumption: \( b^i \leq 1 \)

Agent \( i \)'s portfolio decisions:

\[ \mathbb{E}[r^b | x_i, R, \alpha] \begin{cases} 
> 1 & b^i = 1; \quad m^i = e_1 - 1 \\
= 1 & b^i + m^i + s^i = e_1 \\
< 1 & e_1 = s^i 
\end{cases} \]
Market Clearing

Agents follow monotone threshold strategies: hold bonds & money iff $x_i \geq \hat{x}(R, \alpha)$

\[
\begin{align*}
P(x_i > \hat{x}(R, \alpha)) & \left[ \frac{1}{\text{mass of optimists}} + \frac{(e_1 - 1)}{\text{money demand}} \right] + \frac{\alpha}{\text{AP ratio}} = \frac{S}{\text{random bond supply}} \\
& = \frac{b^{cb}}{m}
\end{align*}
\]

Solving for the cutoff signal:

\[
\hat{x}(R, \alpha) = \theta - \sigma_x \Phi^{-1} \left( \frac{S}{d(\alpha)} \right)
\]

where $d(\alpha) = 1 + \alpha(e_1 - 1) \geq 1$

Effect of asset purchases:

- Market signal truncation and full revelation of $\theta$
- APs select a more optimistic marginal agent
- $\hat{S}'(\alpha) < 0$
Market Signal

\[ \hat{x}(R, \alpha) = \theta - \sigma_x \Phi^{-1} \left( \frac{S}{d(\alpha)} \right) \]
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\[ \text{market signal } \hat{x}(R, \alpha) = z(\theta, S, \alpha) \]

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\[ \alpha_0 = 0 \]
\[ \alpha_1 > 0 \]
\[ \alpha_2 > \alpha_1 \]
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Posterior Beliefs

Market signal
\[ z \mid \theta \sim N(\theta, \sigma_x^2) \quad \text{over } Z = \left[ \theta - \sigma_x \Phi^{-1}(1/d(\alpha)), +\infty \right) \]

“Market” posterior beliefs over \( \theta \)

\[
P(\theta^H|x_i, z, \alpha) = \begin{cases} 
\frac{q}{\sigma_{post}} \phi \left( \frac{\theta^H - x_i + z}{\sigma_{post}} \right) \\
\frac{q}{\sigma_{post}} \phi \left( \frac{\theta^L - x_i + z}{\sigma_{post}} \right) + \frac{1-q}{\sigma_{post}} \phi \left( \frac{\theta^H - x_i + z}{\sigma_{post}} \right) \\
0 
\end{cases}
\]

for \( z \geq z(\theta^H, 1, \alpha) \)

for \( z \in [z(\theta^L, 1, \alpha), z(\theta^H, 1, \alpha)] \)

where \( \sigma_{post}^2 = \sigma_x^2/2 \)

Finally, equilibrium interest rate

\[
p^m(z, \alpha) \frac{1 - \alpha}{1 - R - \alpha} + (1 - p^m(z, \alpha)) \frac{1 - \alpha}{R(1-h) - \alpha} = 1
\]
Effect of asset purchases:
- market signal truncation and full revelation of $\theta \Rightarrow \hat{S}'(\alpha) < 0$
- APs select a more optimistic marginal agent $\Rightarrow \frac{d}{d\alpha} z > 0$
Ex-ante welfare

Integrating over all \((\theta, S)\) realisations

\[
ED(\alpha) = q \int_0^1 \zeta \left( \psi^r \left( z(\theta^H, s, \alpha), \alpha \right) \right) dS \]

\[
(1 - q) \left\{ [1 - \hat{S}(\alpha)] \zeta(1) + \int_0^{\hat{S}(\alpha)} \zeta \left( \psi^d \left( z(\theta^L, s, \alpha), \alpha \right) \right) dS \right\}
\]
Thank You!
$ED(z, \alpha)$

$R(z)$

$f_z(z)$

$\psi(z)$

$p^m(-), p^e(--)$

$ED(\alpha)$
Government Budget Normalisation

\[ \gamma y(\epsilon) = b \]
\[ B_1 \frac{P_1}{P_2} R(1 - \delta h) = \hat{\tau} \]

which becomes

\[ b \frac{P_1}{P_2} R(1 - \delta h) = \tau y(\epsilon) \]
\[ \gamma y(\epsilon) \frac{P_1}{P_2} R(1 - \delta h) = \tau y(\epsilon) \]
\[ \gamma \frac{P_1}{P_2} R(1 - \delta h) = \tau \]
Central Bank Balance Sheet

Central bank balance sheet at $t = 1$

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>bonds $b^{cb}$</td>
<td>money $m$</td>
</tr>
<tr>
<td>storage $s^{cb}$</td>
<td></td>
</tr>
</tbody>
</table>

Central bank balance sheet at $t = 2$

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>bonds $b^{cb} \frac{R(1-\delta h)}{\Pi}$</td>
<td>money $\frac{m}{\Pi}$</td>
</tr>
<tr>
<td>storage $\rho s^{cb}$</td>
<td></td>
</tr>
</tbody>
</table>
Price level determination and real bond returns

Solving for the real return on money

\[
\frac{1}{\Pi} = \rho \frac{1 - \alpha}{1 - \alpha R(1 - \delta h)}
\]

Plug into real bond returns

\[
\psi(R, \alpha, \delta) = \rho \frac{1 - \alpha}{1 - \alpha R(1 - \delta h) - \alpha}
\]

Since \( R \in \left[1, \frac{1}{1-h}\right] \)

- in repayment \( \delta = 0 \) and \( R > 1 \)
  - central bank makes profits
  - there is deflation: \( \frac{1}{\Pi} > \rho \)
  - larger APs imply larger deflation and debt service: \( \uparrow \alpha \Rightarrow \uparrow \psi(0) \)
- in default \( \delta = 1 \) and \( R(1 - h) < 1 \)
  - central bank makes losses
  - there is inflation: \( \frac{1}{\Pi} < \rho \)
  - larger APs imply larger inflation and lower debt service: \( \uparrow \alpha \Rightarrow \downarrow \psi(1) \)