Is Inflation Default? The Role of Information in Debt Crises

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Obs 1: Sovereign Debt and Having Your Currency

- Countries that borrow in their own currency more resilient to debt crises
  - High-debt countries: Japan vs. Italy
  - High-deficit countries: UK vs. Spain

- “Domestic”-currency government bond prices react less to bad news
A Possible Explanation and a Puzzle

- The ability to print money avoids default risk...
  - Interest rates do not jump in anticipation of default

Bassetto and Galli (Chicago Fed, UCL)
Information in Debt Crises
A Possible Explanation and a Puzzle

- The ability to print money avoids default risk...
  ⇒ Interest rates do not jump in anticipation of default
- ...but printing money will cause inflation
  ⇒ Interest rates should jump in anticipation of inflation
Obs 2: Sovereign Spreads vs. Inflation

- Sovereign spreads move very fast, onset of rollover crises is sudden
- Inflation adjusts more slowly (at least in developed economies)
Our Story

- Debt crises require a certain amount of coordination

- With foreign-currency debt, anticipate spike in default spreads
  \Rightarrow \text{coordination among bondholders}

- With domestic-currency debt, anticipate escalation of inflation expectations
  \Rightarrow \text{coordination among price setters}

Price setters less precisely informed about gov't finances
\Rightarrow \text{Information frictions underlie differential response of bond prices to shocks}

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- Price setters less precisely informed about gov’t finances
  \[\Rightarrow\text{Information frictions} \text{underlie differential response of bond prices to shocks}\]
Roadmap

- Stylized macro model
- Show it maps into a two-period Bayesian trading game
  - repeated version of Albagli Hellwig Tsyvinski (2015)
- Comparative statics wrt relevant information precision
Setup and Agents

- Three periods: $t = 1, 2, 3$

- Government (described by a mechanical rule)
  - issues debt in $t = 1$
  - repays (or not) in $t = 3$, depending on fiscal shock $s$

Consequences of fiscal distress

- Euro/foreign currency debt: (explicit) default via haircut
- Yen/domestic currency debt: (implicit) default via inflation

A continuum of two types of agents: “bond traders” and “workers”

- risk neutral, unit wealth, cannot short assets, outside option = stay put

Microfoundations

Bassetto and Galli (Chicago Fed, UCL)
Setup and Agents

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- A continuum of two types of agents: “bond traders” and “workers”
  - risk neutral, unit wealth, cannot short assets, outside option $=$ stay put
Government auctions debt at price $q_1$, promised repayment $\hat{s}(q_1)$;

Examples:

- Eaton and Gersowitz (1981): $\hat{s}(q_1) \equiv \hat{s}$
- Calvo (1988): $\hat{s}(q_1) \equiv \hat{s}/q_1$

Bond traders

- buy bonds conditional on price $q_1$ (or stay put)
- info on $s$: prior $N(\mu_0, 1/\alpha_0)$, private signal $x_{i,1} \sim N(s, 1/\beta_1)$

Residual noise-traders demand $\Phi(\epsilon_1)$, with $\epsilon_1 \sim N(0, 1/\psi_1)$
Timing and Actions – Second Period

- Bond traders must offload bonds
  - to new bond traders (€), or to workers through cash (¥)
- New bond traders (or workers)
  - buy bonds (cash) conditional on price $q_2$ (or stay put)
  - info on $s$: prior $N(\mu_0, 1/\alpha_0)$, private signal $x_{i,2} \sim N(s, 1/\beta_2)$
- Residual noise-agents demand $\Phi(\epsilon_2)$, with $\epsilon_2 \sim N(0, 1/\psi_2)$
Timing and Actions – Third Period

- If $s \geq \hat{s}(q_1)$, govt repays debt
- If $s < \hat{s}(q_1)$, default (or inflation) $\Rightarrow$ haircut (or currency debasement) $1 - \theta$
Timing and Actions – Third Period

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Symmetries and key difference:
- Same eventual default/inflation payoff at the end ($t = 3$)
- Same primary-market participants at the start ($t = 1$)
- Identity of secondary-market ($t = 2$) participants different:
  - bond traders (€) better informed than workers (¥)
  $\Rightarrow \beta_2$ (or $\psi_2$) higher under €
Three Cases

1. No recall of past prices + exogenous default threshold \( \hat{s} \)
2. Recall of past prices + exogenous default threshold \( \hat{s} \)
3. Recall of past prices + endogenous default threshold \( \hat{s}(q_1) \)
The Simplest Case

Assume

- $\hat{s}(q_1) \equiv \hat{s}$ (constant)
- period-2 agents do not observe $q_1$
The Simplest Case

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- period-2 agents do not observe $q_1$

Period-\(t\) agents’ information set

- prior
- private signal $x_{i,t}$
- can condition on current-period price $\Rightarrow$ demand schedules $d(x_{i,t}, q_t)$
Period-2 Agents: Payoffs and Strategies

- **Expected payoff**

\[ \theta \cdot \text{Prob}(s < \hat{s}|x_{i,2}, q_2) + 1 \cdot \text{Prob}(s \geq \hat{s}|x_{i,2}, q_2) - q_2 \]

- (€) \( \mathbb{E}_{i,2}[\text{bond repayment}] \)
- (¥) \( \mathbb{E}_{i,2}[1/P_3] \)
- bond price \( 1/P_2 \)
Period-2 Agents: Payoffs and Strategies

- Expected payoff

\[ \theta \cdot \text{Prob}(s < \hat{s} | x_{i,2}, q_2) + 1 \cdot \text{Prob}(s \geq \hat{s} | x_{i,2}, q_2) - q_2 \]

\[ (€) \quad E_{i,2}[\text{bond repayment}] \]

\[ (¥) \quad E_{i,2} [1/P_3] \]

- Posterior beliefs on \( s \) are FOSD-increasing in \( x_{i,2} \)
  - Buy if signal is above threshold:

\[ d(x_{i,2}, q_2) = \mathbb{1}[x_{i,2} \geq \hat{x}_2(q_2)] \]
Period-2: Market Clearing and Beliefs

- Period-2 market clearing condition

\[
\text{Prob}(x_{i,2} \geq \hat{x}_2(q_2)|s) = 1 - \Phi(\epsilon_2)
\]

\(s + \frac{\epsilon_2}{\sqrt{\beta_2}} = \hat{x}_2(q_2)\)

- Market clearing implies

informed nominal-asset demand \quad \text{nominal-asset supply (net of noise agents)}
Period-2: Market Clearing and Beliefs

- **Period-2 market clearing condition**

\[
\text{Prob}(x_{i,2} \geq \hat{x}_2(q_2)|s) = 1 - \Phi(\epsilon_2)
\]

\[
\text{informed nominal-asset demand} \quad \text{nominal-asset supply (net of noise agents)}
\]

- **Market clearing implies**

\[
z_2 := s + \frac{\epsilon_2}{\sqrt{\beta_2}} = \hat{x}_2(q_2)
\]

- **We focus on equilibria where** \(z_t\) **is informationally equivalent to** \(q_t\)

- **Second-period agents posterior beliefs**

\[
s|x_2, z_2 \sim N\left(\frac{\alpha_0 \mu_0 + \beta_2 x_2 + \beta_2 \psi_2 z_2}{\alpha_0 + \beta_2 (1 + \psi_2)}, \frac{1}{\alpha_0 + \beta_2 (1 + \psi_2)}\right)
\]
Period-2: Equilibrium

- Marginal agent’s indifference condition

\[ \theta + (1 - \theta) \text{Prob}(s \geq \hat{s}|x_{i,2} = \hat{x}_2(q_2), q_2) = q_2 \]

- Equilibrium $t = 2$ price

\[ q_2(z_2) = \theta + (1 - \theta) \Phi \left( \frac{(1 - w_S)\mu_0 + w_S z_2 - \hat{s}}{\sigma_S} \right) \]

\[ w_S := \frac{\beta_2(1+\psi_2)}{\alpha_0 + \beta_2(1+\psi_2)}, \quad \sigma_S^2 := \frac{1}{\alpha_0 + \beta_2(1+\psi_2)} \]
Comparative Statics (more precise info \(= \) higher \(\beta_2\) or \(\psi_2\))

\[
q_2(z_2)
\]

- Euro economy (more precise \(t = 2\) info)
- Yen economy (less precise \(t = 2\) info)
Period-1: Strategies and Beliefs

- Expected payoff

\[ \mathbb{E}[q_2(z_2)|x_{i,1}, q_1] - q_1 \]

- Monotone threshold strategies again

- Market clearing implies

\[ z_1 := s + \epsilon_1 / \sqrt{\beta_1} = \hat{x}_1(q_1) \]
Period-1: Strategies and Beliefs

- Expected payoff
  \[ \mathbb{E}[q_2(z_2)|x_{i,1}, q_1] - q_1 \]

- Monotone threshold strategies again

- Market clearing implies
  \[ z_1 := s + \frac{\epsilon_1}{\sqrt{\beta_1}} = \hat{x}_1(q_1) \]

- First-period agents posterior beliefs on \( z_2 \), not just \( s \)
  \[ z_2|(z_1, x_1) \sim N \left( \frac{\alpha_0 \mu_0 + \beta_1 x_1 + \beta_1 \psi_1 z_1}{\gamma_1}, \frac{1}{\gamma_1} + \frac{1}{\psi_2 \beta_2} \right) \]
Period-1: Equilibrium

Marginal traders’ indifference condition

$$\mathbb{E}[q_2(z_2) | x_{i,1} = \hat{x}_1(q_1), q_1] = q_1$$
Period-1: Equilibrium

- Marginal traders’ indifference condition

\[ \mathbb{E}[q_2(z_2)|x_{i,1} = \hat{x}_1(q_1), q_1] = q_1 \]

- Equilibrium \( t = 1 \) price

\[ q_1(z_1) = \theta + (1 - \theta)\Phi \left[ \frac{\mu_0 - \hat{s}}{\sqrt{w_S^2\sigma_{S|B}^2 + \sigma_S^2}} + \frac{w_S w_B}{\sqrt{w_S^2\sigma_{S|B}^2 + \sigma_S^2}} (z_1 - \mu_0) \right] \]

\[ w_B := \frac{\beta_1(1+\psi_1)}{\alpha_0 + \beta_1(1+\psi_1)} \], \quad \sigma_{S|B}^2 := \frac{1}{\gamma_1} + \frac{1}{\psi_2\beta_2} \]
Comparative Statics (more precise info $\Rightarrow$ higher $\beta_2$ or $\psi_2$)

Propositions 1&2
What if there is Recall of the First-Period Price?

- Same payoffs, different information set for period-2 agents
- $q_1$ new source of common knowledge with period-1 traders
- $q_1 \iff z_1$
- Marginal period-1 trader and period-2 trader have different weight on $z_1$; period-2 information is not finer than period 1
- Difference breaks law of iterated expectations:

$$q_1 = E[E[\pi(\theta)|I_2]|I_1]$$
Second-period agents posterior beliefs

\[ s | x_2, z_2, z_1 \sim N \left( \frac{\alpha_0 \mu_0 + \beta_1 \psi_1 z_1 + \beta_2 x_2 + \beta_2 \psi_2 z_2}{\alpha_0 + \beta_1 \psi_1 + \beta_2 (1 + \psi_2)}, \sigma^2_S := \frac{1}{\alpha_0 + \beta_1 \psi_1 + \beta_2 (1 + \psi_2)} \right) \]
Second-period agents posterior beliefs

\[ s|x_2, z_2, z_1 \sim N \left( \frac{\alpha_0 \mu_0 + \beta_1 \psi_1 z_1 + \beta_2 x_2 + \beta_2 \psi_2 z_2}{\alpha_0 + \beta_1 \psi_1 + \beta_2 (1 + \psi_2)}, \frac{1}{\alpha_0 + \beta_1 \psi_1 + \beta_2 (1 + \psi_2)} \right) \]

Equilibrium \( t = 1 \) price

\[ q_1(z_1) = \theta + (1 - \theta) \Phi \left[ \frac{\mu_0 - \hat{s}}{\sqrt{w_2, S \sigma_{S|B}^2 + \sigma_S^2}} + \frac{(w_{1,S} + w_{2,S} w_B)}{\sqrt{w_2, S \sigma_{S|B}^2 + \sigma_S^2}} (z_1 - \hat{s}) \right] \]
Comparative Statics: Some Intuition

\[ q_1(z_1) = \theta + (1 - \theta)\Phi \left[ \frac{\mu_0 - \hat{s}}{S} + K(z_1 - \mu_0) \right] \]

\[ K := \frac{(w_{1,s} + w_{2,s}w_B)}{\sqrt{w_{2,s}^2 \left( \frac{1}{\gamma_1} + \frac{1}{\beta_2\psi_2} \right) + \sigma_s^2}} \]

- Always get single crossing, as before
- Direction of crossing dictated by \( K \)
Comparative Statics: Some Intuition

\[ q_1(z_1) = \theta + (1 - \theta)\Phi \left[ \frac{\mu_0 - \widehat{s}}{S} + K(z_1 - \mu_0) \right] \]

\[ K := \frac{(w_1, s + w_2, s w_B)}{\sqrt{w_2, s \left( \frac{1}{\gamma_1} + \frac{1}{\beta_2 \psi_2} \right) + \sigma_s^2}} \]

- Always get single crossing, as before
- Direction of crossing dictated by \( K \)
- Effect of \( \beta_2, \psi_2 \) on \( K \) more involved:
  - \( \beta_2 \uparrow \implies \) period-2 agents give less weight to prior, but also to \( q_1 \)
  - Less weight on prior \( \implies \) \( q_2 \) tracks \( s \) better
  - Less weight on \( q_1 \implies \) \( q_2 \) tracks \( s \) better, but potentially less correlated with \( q_1 \), ambiguous
Comparative Statics

\[ K := \frac{\left( w_{1,S} + w_{2,S}w_B \right)}{\sqrt{w_{2,S}^2\sigma^2_{S|B} + \sigma^2_{S}}} \]
Single Crossing Again

\[ q_1(z_1) \]

- **Euro economy (more precise \( t = 2 \) info)**
- **Yen economy (less precise \( t = 2 \) info)**

\[ (\hat{z}_1, q_1(\hat{z}_1)) \]
Endogenous Default Threshold: Equilibrium

- Consider endogenous default cutoff: gov’t repays iff $s \geq \hat{s}(q_1)$
Consider endogenous default cutoff: gov’t repays iff \( s \geq \hat{s}(q_1) \)

Period-1 price only implicitly characterized, solves

\[
q_1 = \theta + (1 - \theta) \Phi \left[ \frac{\mu_0 - \hat{s}(q_1)}{\sqrt{w_2,S \sigma_S^2|B} + \sigma_S^2} + \frac{(w_1,S + w_2,S w_B)}{\sqrt{w_2,S \sigma_S^2|B} + \sigma_S^2} (z_1 - \mu_0) \right]
\]
Endogenous Default Threshold: Comparative Statics

Comparative statics

- on $\psi_2$: still valid, single crossing
Endogenous Default Threshold: Comparative Statics

Comparative statics

- on $\psi_2$: still valid, single crossing
- on $\beta_2$: price changes are still the same in tail events

\[ q_1(z_1) \]

Euro economy (more precise $t = 2$ info)
Yen economy (less precise $t = 2$ info)
$(z_1, q_1(z_1))$
Conclusion

- Heterogeneity of information has important implications for debt management
- We have shown insurance role of domestic-currency debt
- Next step: optimal theory of currency denomination (study of effects on ex ante price)
Thank You!
The “Original Sin”

- Some countries seem to be unable to issue domestic debt
- Perhaps because of time-inconsistency (Calvo, 1989)
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- Bordo-Meissner (2006): Currency mismatch not necessarily associated with more frequent crises
The “Original Sin”

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- If this were the problem, we would expect interest rates to be more sensitive to bad news with domestic-currency debt
- Bordo-Meissner (2006): Currency mismatch not necessarily associated with more frequent crises
- Ability to devalue and mitigate recession not always relevant (in the 2008 crisis the yen appreciated)
Macro Model: Setup and actors

- Three periods
- Bond traders: strategic and noise
- Workers: strategic and noise
- Government (described by a mechanical rule)
Workers: Preferences and Technology

- Only alive in periods 2 and 3

- Strategic workers
  - One unit of endowment in period 2
  - Wish to consume in period 3, risk neutral
  - Can store good (zero return) or sell it

- Noise workers
  - (Unobserved) relative mass $\Phi(\epsilon_2^w), \epsilon_2^w \sim N(0, 1/\psi_2^w)$
  - Can produce in period 3
  - Demand 1 unit of consumption in period 2
Bond Traders: Preferences and Technology

- 2 OLGs living for two periods
- Endowed with goods when young
- Want to consume when old, risk neutral

- Strategic traders:
  ▶ Can store
  ▶ Can buy one unit of government bonds

- Noise traders:
  ▶ Demand an (unobserved) fraction $\Phi(\epsilon_t^b)$, $\epsilon_t^b \sim N(0, 1/\psi_t^b)$, of gov’t debt

- Mass of bond traders negligible compared to workers
Government - “Euro” scenario

- Auctions one unit of debt in period 1 (per capita per young strategic trader), price $q_1$

- Debt is a promise to pay $\hat{s}(q_1)$ Euros (goods) in period 3. Examples:
  - $\hat{s}(q_1) \equiv 1$ (Eaton and Gersovitz)
  - $\hat{s}(q_1) \equiv 1/q_1$ (Calvo)

- In period 3, gov’t collects taxes, depending on the realization of $s \sim N(\mu_0, 1/\alpha_0)$:
  - If $s \geq \hat{s}(q_1)$, full repayment
  - Otherwise, haircut $1 - \theta$, gov’t pays back $\theta \hat{s}(q_1)$
## Euro Markets

<table>
<thead>
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<th></th>
<th>$t$</th>
<th>Govt</th>
<th>$BT_1$</th>
<th>$BT_2$</th>
<th>SW</th>
<th>NW</th>
<th>Price</th>
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<td>1</td>
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<td>$q_1 = E_{BT_1}[q_2]$</td>
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<td></td>
<td></td>
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<td>$q_2 = E_{BT_2}[q_3]$</td>
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<tr>
<td>3</td>
<td></td>
<td>repay</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$q_3 = \delta \theta + (1 - \delta)$</td>
</tr>
</tbody>
</table>

$e_1$ and $e_2$ are goods; $c_2$ is a bond; $c_3$ is storage (dashed)

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**Yen Markets**

Bassetto and Galli (Chicago Fed, UCL)
Government - “Yen” scenario

- Auctions one unit of debt in period 1 (per capita per young strategic trader), price $q_1$
- Debt is a promise to pay $\hat{s}(q_1)$ Yen.

- In period 2, gov’t prints Yen, pays debt back.

- In period 3, gov’t collects taxes, depending on the realization of $s \sim N(\mu_0, 1/\alpha_0)$:
  - If $s \geq \hat{s}(q_1)$, collects $\hat{s}(q_1)$
  - Otherwise, collects $\theta \hat{s}(q_1)$

(same as Euro scenario)

- Period-3 taxes used to buy Yen back. Price level is either 1 or $1/\theta$. 

Price Level Determination
Yen Markets

<table>
<thead>
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<td>$c_2$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>buy back</td>
<td>$c_3$</td>
<td>$e_3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
q_1 = \mathbb{E}_{BT_1} \left[ \frac{1}{P_2} \right]
\]

\[
\frac{1}{P_2} = \mathbb{E}_{SW} \left[ \frac{1}{P_3} \right]
\]

\[
\frac{\hat{s}(q_1)}{P_3} = [\delta \theta + (1 - \delta)]\hat{s}(q_1)
\]

Goods; bonds; cash; storage (dashed)
Euro vs. Yen: the Key Difference

- Eventual default/inflation is the same at the end \((t = 3)\)
- Identity of primary-market participants is the same at the start \((t = 1)\)

- **Period 2** Identity of secondary-market participants different:
  - Under Euro, bonds offloaded to **new bond traders**
  - Under Yen, bonds offloaded to **workers (through cash)**

- With same information, same prices/payoffs in the 2 scenarios:
  - collapse them into a single problem: \(q_2 := 1/P_2\) in the Yen case
  - index scenarios with period-2 agents’ information precision
Government money valuation equation

\[ \frac{M}{P_3} = \text{real tax revenues} \]

and since \( M = \hat{s}(q_1) \) (govt repays debt with money at \( t = 2 \))

\[ \frac{M}{P_3} = \frac{\hat{s}(q_1)}{P_3} = \delta \cdot \theta \hat{s}(q_1) + (1 - \delta) \cdot \hat{s}(q_1) \]

so that

\[ \begin{align*}
\delta &= 1 & P_3 &= 1/\theta \\
\delta &= 0 & P_3 &= 1
\end{align*} \]
A Perfect Bayesian Equilibrium consists of bidding strategies \( d(x_{i,t}, q_t) \) for strategic players, a price function \( q(s, \epsilon_t) \) and posterior beliefs \( p(x_{i,t}, q_t) \) such that

(i) \( d(x_{i,t}, q_t) \) is optimal given beliefs \( p(x_{i,t}, q_t) \),

(ii) \( q(s, \epsilon_t) \) clears the market for all \( (s, \epsilon_t) \), and

(iii) \( p(x_{i,t}, q_t) \) satisfies Bayes’ Law for all market clearing prices \( q_t \).
More Definitions

- Precision of first-period posterior beliefs

\[
\frac{1}{\gamma_1} := \frac{1}{\alpha_0 + \beta_1(1 + \psi_1)}
\]

Case 1: \( t = 1 \) beliefs

Case 1: comparative statics

- Second-period Bayesian weights (case with recall)

\[
w_{1,S} := \frac{\beta_1 \psi_1}{\alpha_0 + \beta_1 \psi_1 + \beta_2(1 + \psi_2)} \quad w_{2,S} := \frac{\beta_2(1 + \psi_2)}{\alpha_0 + \beta_1 \psi_1 + \beta_2(1 + \psi_2)}
\]

Case 2: \( q_1 \)

- Aggregate noise term of first-period price (case with recall)

\[
S := \sqrt{w_{2,S}^2 \left( \frac{1}{\gamma_1} + \frac{1}{\beta_2 \psi_2} \right) + \sigma^2_S}
\]

Case 2: \( q_1 \)  
Case 2: comparative statics
Simplest Case

Proposition (1)
There exists a cutoff level $\hat{z}_1^\beta \in \mathbb{R}$ such that when $z_1 < \hat{z}_1^\beta$, a decrease in $\beta_2$ improves the issuance price $q_1$, whereas the reverse occurs for $z_1 > \hat{z}_1^\beta$.

Proposition (2)
There exists a cutoff level $\hat{z}_1^\psi \in \mathbb{R}$ such that when $z_1 < \hat{z}_1^\psi$, a decrease in $\psi_2$ improves the issuance price $q_1$, whereas the reverse occurs for $z_1 > \hat{z}_1^\psi$.
### Proposition (3)

There exists a cutoff level $\hat{z}_1^\psi \in \mathbb{R}$ such that when $z_1 < \hat{z}_1^\psi$, a decrease in $\psi_2$ improves the issuance price $q_1$, whereas the reverse occurs for $z_1 > \hat{z}_1^\psi$.

### Proposition (4)

Assume that $\psi_2 \geq \psi_1$ and $\beta_2^A \geq \beta_1$. Let $\beta_2^B < \beta_2^A$. Then there exists a cutoff level $\hat{z}_1^\beta \in \mathbb{R}$ such that when $z_1 < \hat{z}_1^\beta$, $q_1$ evaluated at $\beta_2^A$ is smaller than at $\beta_2^B$, whereas the reverse occurs for $z_1 > \hat{z}_1^\beta$, holding all other parameters fixed.
With Recall, Endogenous Threshold

Proposition (5)

Assume that $\psi_2 \geq \psi_1$ and $\beta_2^A \geq \beta_1$, and let the conditions for equilibrium uniqueness hold. Let $\beta_2^B < \beta_2^A$. Then there exist two cutoffs level $\hat{z}_1^L \leq \hat{z}_1^H \in \mathbb{R}$ such that when $z_1 < \hat{z}_1^L$, $q_1$ evaluated at $\beta_2^A$ is smaller than at $\beta_2^B$, whereas the reverse occurs for $z_1 > \hat{z}_1^H$, holding all other parameters fixed.