Self-Fulfilling Debt Crises and Government Policy

Carlo Galli
Role for self-fulfilling beliefs in sovereign default models

- Motivated by emerging markets experience and Eurozone crisis
- Bond spreads high and volatile...
- ...but often disconnected to fundamentals and actual defaults
- EZ debt crisis: high spreads as bad equilibrium, motivation for OMT
Self-Fulfilling Debt Crises and Multiple Equilibria

Role for self-fulfilling beliefs in sovereign default models

- Motivated by emerging markets experience and Eurozone crisis
- Bond spreads high and volatile...
- ...but often disconnected to fundamentals and actual defaults
- EZ debt crisis: high spreads as bad equilibrium, motivation for OMT

Link between spreads, gov’t policy and fundamentals important

- Two-way empirical relationship between country spreads and business cycle
  [Neumeyer-Perri (2005), Uribe-Yue (2006)]
- Austerity policies *in response to* EZ crisis (Italy, Spain)
- Micro evidence of gov’t spreads pass-through to investment, output
  [Arellano et al. (2017), Bocola (2016), Bottero et al. (2017)]
This Paper

Standard sovereign default model, with endogenous output

- Circular feedback: spreads ⇔ govt debt ⇔ domestic policy (gov’t investment)
- Non-contractible gov’t policy
- Austerity induced by debt crises can generate belief-driven equilibria
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Debt crisis mechanism

- confidence crisis ⇒ higher spreads, costlier to borrow
- ⇒ govt raises less funds, cuts down on consumption and investment instead
- ⇒ growth ↓, future default incentives ↑ ⇒ pessimistic expectations verified
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Model properties

- multiplicity mechanism: dynamic, real effect of spreads
- debt overhang dynamics: ↑ debt may ↓ investment incentives
- crisis periods (bad eqm) may feature large & finite spreads, lower debt (consistent with Aguiar et al. (2016))
Outline

1. Stylised 2-period model
   - highlight multiplicity mechanism
   - characterize equilibria
   - (appendix) analytical proof for deterministic case

2. Infinite-horizon numerical example
   - show quantitative properties
   - static vs dynamic multiplicity
Government Problem

- Two periods, $t = 0, 1$
- Government born with $w := f(k_0) - b_0$, solves

$$V(w) = \max_{c_0, c_1^R, c_1^D} U(c_0) + \int \max_{R, D} \left\{ U(c_1^R), U(c_1^D) \right\} dG(\gamma)$$

s.t. $c_0 = w + qb - k$
$c_1^R = f(k) - b$
$c_1^D = f(k)\gamma$

- Govt cannot commit to either $k$ or repay
- If default, production loss $(1 - \gamma)$, with $\gamma \sim G(0, 1)$
- Repay iff default costs are high: $1 - \gamma \geq \frac{b}{f(k)}$
- Discount bonds, perfectly patient lenders $\Rightarrow$ risk-free debt price $= 1$
Timing in $t = 0$

1. Government chooses debt issuance $b$

2. Lenders pay price $q$, government raises $qb$ resources

3. Consumption/Investment chosen after debt issuance, taking $(q, b)$ as given
   - objective function for investment, given $(w, q, b)$
     \[
     W(k; w, q, b) = u(w + qb - k) + \int \max_{R,D} \{u[f(k) - b], u[f(k)\gamma]\} \; dG(\gamma)
     \]
   - $k^*(w, q, b)$ unique solution to $\max_k W(k; w, q, b)$
   - consumption determined residually
Lenders’ Problem

Lenders are atomistic, perfectly competitive ⇒ make zero-profits in expectations

- must anticipate government’s investment strategy $k^*$

Set of zero-profit prices at which lenders are willing to buy $b$

$$Q(w, b) = \left\{ q : q = \text{Prob}\left( (1 - \gamma) \geq \frac{b}{f[k^*(w, q, b)]} \right) \right\}$$

⇒ Calvo timing setup: repay if $y' - b'R \geq \gamma y' \Rightarrow (1 - \gamma) \geq \frac{b'R}{y'}$

$Q(w, b)$ may be a correspondence for some values of $(w, b)$
Equilibrium

Collection of

- government policies $b_i^*(w)$, $k_i^*(w)$, value functions $V_i(w)$
- creditors debt price schedules $q_i(w, b)$

such that

- $b_i^*(w)$ and $k_i^*(w)$ solve the government’s problem and achieve $V_i(w)$, conditional on $q_i$
- given government policies, price functions $q_i(w, b)$ satisfy lenders’ zero-profit condition for all $b$
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Now:
1. show conditions for existence of multiple debt price schedules
2. show conditions & states for multiple equilibria
When is $Q(w, b)$ a Correspondence?

Given state $w$

Set of Zero-Profit Prices
When is $Q(w, b)$ a Correspondence?

Given state $w$

- ignore unstable part of schedule, split into single-valued fns $q_i(w, b)$
- assume government observes $i$ before issuing debt
  $\approx$ observing secondary market conditions
When is $Q(w, b)$ a Correspondence?

Fix $w, b$

$\text{Prob}\left(1 - \gamma \geq \frac{b}{f[k^*(w,q,b)]}\right)$

$k^*(w,q,b) = \frac{w + qb}{w+q}$
When is $Q(w, b)$ a Correspondence?

Fix $w, b$

\[
\text{Prob}\left(1 - \gamma \geq \frac{b}{f[k^*(w, q, b)]}\right)
\]

\[
k^*(w, q, b) = \frac{w + qb}{w + q}
\]
Equilibrium Policy: High Endowment

- There might exist multiple schedules...
- ...but does the government ever select them?
Equilibrium Policy: Low Endowment

- There might exist multiple schedules...
- ...but does the government ever select them?
When govt policy is risk-free

- \( f'(k_u) = 1 \) (MPK = return on savings/cost of borrowing)
- \( b_u(w) = \frac{f(k_{rf}) + k_{rf} - w}{2} \) (when feasible, first-best)
Multiple Equilibria

Gov’t Utility

Initial Wealth $w$

Investment

Initial Wealth $w$

Debt/GDP

Initial Wealth $w$

Debt Price

Initial Wealth $w$
Summing Up

Confidence crisis

- more expensive to borrow, tighter govt budget set
- cut borrowing, consumption & investment (≈ raise taxes)
- debt/GDP↓, but lower utility and depressed output

Here “austerity” is bad but necessary

- fiscal tightening to avoid high (extreme here) borrowing costs
- not desirable, but only alternative during crisis
Infinite Horizon Numerical Example

• 1 period = 1 quarter

• Default causes
  • random iid production loss $\gamma$, permanent
  • permanent exclusion from debt markets

• Qualitative predictions very similar to 2-period model
Value Functions

Start-of-period value function:

\[ V(k, b, \gamma) = \max_{R,D} \{ V^R[f(k) - b], V^D(k, \gamma) \} \]

Repay value function:

\[ V^R[f(k) - b] = \max_{k', b'} u[w + q(w, b')b' - k'] + \beta \sum_{\gamma} P(\gamma) V(k', b', \gamma) \]

Default value function:

\[ V^D(k, \gamma) = \max_{k'} u[\gamma f(k) - k'] + \beta V^D(k', \gamma) \]

Debt price correspondence

\[ Q(w, b') = \left\{ q : q = \frac{1}{R} \sum_{\gamma} P(\gamma) \mathbb{1} \left[ V^R[f(k^*) - b'] \geq V^D(k^*, \gamma) \right] \right\} \]

\[ k^* := k^*(w, q, b') \]
• Debt price schedule is still a correspondence
• To coordinate lenders’ beliefs, iid sunspot

\[ i = \begin{cases} 
G \quad \text{w.p. } \pi & \rightarrow \quad V_G[f(k) - b], Q_G(w, b') \\
B \quad \text{w.p. } 1 - \pi & \rightarrow \quad V_B[f(k) - b], Q_B(w, b')
\end{cases} \]
Debt Price Function Example

Debt Price Schedules for \( w = 0.23 \times f(k_u) \)

- **Good schedule,** \( q_G(w, b) \)
- **Bad schedule,** \( q_B(w, b) \)
Introduction

2-Period Model

Numerical Example

Policies and Equilibrium Prices

Eqm Wealth L.O.M. ($f(k_u)$)

\[ w'_{G}(w) \]

\[ w'_{B}(w) \]

\[ w_{risk-free} \]

Policies ($f(k_u)$)

\[ b^*_{G}(w) \]

\[ k^*_{G}(w) \]

\[ w_{risk-free} \]

Eqm Debt Prices $q[b^*_i(w), w]$
“Secondary Mkt” Spreads and Default Probabilities

Secondary Mkt Spreads pre-z shock

Default Probs by \((k, b)\) \((\%f(k_u))\)
What about “dynamic” multiplicity?

So far, “static” multiplicity: given $E[V(k', b', \gamma)]$, self-confirming beliefs over $k'$ today

- Limitation? In bad eqm no risky borrowing, “endogenous austerity”

Dynamic, circular mechanism typical of sovereign default models:

- $V^R \to$ default cutoff $\hat{b}(k, \gamma)$ via $V^R[f(k) - \hat{b}] = V^D[\gamma f(k)]$
- cutoff $\hat{b}(k, \gamma) \to$ price fn $Q(w, b')$
- price fn $Q(w, b') \to V^R$
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- $V^R \rightarrow$ default cutoff $\hat{b}(k, \gamma)$ via $V^R[f(k) - \hat{b}] = V^D[\gamma f(k)]$
- cutoff $\hat{b}(k, \gamma) \rightarrow$ price fn $Q(w, b')$
- price fn $Q(w, b') \rightarrow V^R$

Is mechanism strong enough to generate multiple $(V^R, \hat{b}, Q)$ triplets?

- No, in canonical Eaton-Gersovitz models (Auclert-Rognlie (2016))
- Answer seems different here with endogenous income

Different beliefs on continuation values

- parallel debt schedules
- more realistic equilibrium prices?
Deterministic Example: Dynamic Multiplicity

\[ V^R [f(k) - b] \]

\[ \hat{b}(k) \]

\[ q(w, b') \]
Conclusion

• Real, dynamic effect of spreads in standard sovereign default model
• Two-way feedback between spreads, policy and real activity
  ⇒ beliefs interact with both debt and domestic policy

Extensions

• add private sector and govt taxation → feedback spreads-govt-pvt sector
• dynamic multiplicity in infinite horizon
Appendix

Appendix

Deterministic Example with Analytical Characterization
Related Literature

Problems tackled separately in the literature:

  - (fundamentals, policy) $\to$ spreads

- **Lending channel** [Bocola (2016), Arellano et al. (2017), Ari (2017), Balke (2017), Bottero et al. (2017)]
  - spreads $\to$ fundamentals (via banking sector)

- **Austerity policies** [Arellano-Bai (2016), Conesa-Kehoe-Ruhl (2017)]
  - spreads $\to$ fundamentals (via tax policy)

  - spreads $\leftrightarrow$ debt policy (no fundamentals)
Pricing Equations Review

**PR := Probability of Repayment**

- issue $b'$, get price $q$, repay tmr if
  $$y' - b' \geq h(y') \quad \Rightarrow \quad q = PR_y[b']$$
- off-equilibrium, adjust $c$

Government as price-taker (Lorenzoni-Werning (2014))
- given $q$, issue $b'$, repay if
  $$y' - b' \geq h(y') \quad \Rightarrow \quad q = PR_y[b'(q)]$$
- off-equilibrium, adjust $b'$

Calvo (1988) timing
- issue $b'$ at interest rate $1/q$, repay if
  $$y' - b' \frac{1}{q} \geq h(y') \quad \Rightarrow \quad q = PR_y[b' \frac{1}{q}]$$

If output is endogenous: $y' = \mathcal{H}(q, b', \cdot) \leftrightarrow$ This paper:
$$y' = f[k^*(w, q, b')]$$
- issue $b'$, get $q$, debt price $q = PR_y[b', \mathcal{H}(q, b', \cdot)]$
  - same timing/commitment of Eaton-Gersovitz framework
  - $\mathcal{H}$ can be many things
First-Order Conditions

- Govt repays iff \( \gamma \leq \hat{\gamma} := 1 - \frac{b}{f(k)} \)
- Define debt price schedule as \( q_i(w, b) \)
- Capital FOC:
  \[
  u'(c_0) = \beta f'(k) \left[ G(\hat{\gamma})u'(c_R) + \int_{\hat{\gamma}}^{\gamma} \gamma u'(c_D) dG(\gamma) \right]
  \]
- Debt FOC:
  \[
  u'(c_0) = \beta \frac{1}{q_i(w, b) + \frac{\partial q_i(w, b)}{\partial b} b} G(\hat{\gamma})u'(c_R)
  \]
- When debt is risk-free:
  \[
  q_i(w, b') + \frac{\partial q_i(w, b')}{\partial b'} b' = \frac{1}{R} \quad \text{and} \quad G(\hat{\gamma}) = 1 \quad \Rightarrow \quad \begin{cases} 
  f'(k) = R \\
  u'(c_0) = \beta Ru'(c)
  \end{cases}
  \]
When is $Q(w, b)$ a Correspondence?

$\text{Prob} \left( 1 - \gamma \geq \frac{b}{f[k^*(w,q,b)]} \right)$

- stdev*0.5
- stdev*1.0
- stdev*1.5

\[
\frac{k^*(w,q,b)}{w+qb}
\]
Investment Objective Fn

- Examine the investment decision, keeping everything else \((w, q, b)\) fixed

\[
W(k; w, q, b) = u(w + qb - k) + \max \{u[f(k) - b], u[f(k)\gamma]\}
\]

- \(k_1(b) := \text{lowest } k \text{ s.t. govt repays } b\)
- \(W_p(k; w, q, b)\) is obj. fn. assuming govt will repay w.p. \(p\)
When is $Q(w, b)$ a Correspondence? Sufficient Conditions

\[ k^*(w, 0, b) < k_1(b) \iff 0 \in Q(w, b) \iff W'_1(k_1(b); w, 0, b) \leq 0 \]
When is $Q(w, b)$ a Correspondence? Sufficient Conditions

$k^*(w, 0, b) < k_1(b) \iff 0 \in Q(w, b) \iff W_1'(k_1(b); w, 0, b) \leq 0$

$k^*(w, 1, b) \geq k_1(b) \iff 1 \in Q(w, b) \iff W_0'(k_1(b); w, 1, b) \geq 0$
When is $Q(w, b)$ a Correspondence? Characterization

- For each state $w$, characterize debt levels such that $(0, 1) \in Q(w, b)$
- for all $b \in [b(w), \overline{b}(w)]$, there are multiple zero-profit prices
Sufficient Condition

**Proposition**

*Given state* \( w \), *if*

\[
\frac{u'(w + b(w) - k_1[b(w)])}{u'(w - k_1[b(w)])} \leq \gamma
\]

*then*

\( b(w) \leq \bar{b}(w) \)  and  \( (0, 1) \in Q(w, b) \quad \forall b \in [b(w), \bar{b}(w)] \)
Sufficient Condition

Proposition

Given state \( w \), if

\[
\frac{u'(w + b(w) - k_1[b(w)])}{u'(w - k_1[b(w)])} \leq \gamma
\]

then

\( b(w) \leq \bar{b}(w) \) and \((0, 1) \in Q(w, b)\) \( \forall b \in [b(w), \bar{b}(w)] \)

In words, there exist multiple zero-profit prices if \( k^* \)

- implies default when \( q = 0 \)
  - low auction revenues, high \( u'(c_0) \) \( \rightarrow \) \( \text{MC(risk-free } k) \gg \text{MB(risk-free } k) \)
- is risk-free when \( q = 1 \)
  - high auction revenues, low \( u'(c_0) \) \( \rightarrow \) \( \text{MC(default } k) \ll \text{MB(default } k) \)
Equilibrium Policy: High Endowment

- There might exist multiple schedules...
- ...but does the government ever select them?
Equilibrium Policy: Low Endowment

- There might exist multiple schedules...
- ...but does the government ever select them?
Unconstrained Risk-Free Policy

When govt policy is risk-free

- \( f'(k_u) = 1 \) (MPK = return on savings/cost of borrowing)
- \( b_u(w) = \frac{f(k_{rf}) + k_{rf} - w}{2} \) (when feasible, Pareto-efficient)

If \( 1 \notin Q(w, b_u(w)) \), policy is not feasible!

- \( \Rightarrow \) depends on schedule
- if so, govt is borrowing constrained
- borrowing constraint depends on schedule
Multiple Equilibria

\([w, \overline{w}]\) where \(k_u, b_u(w)\)

- are feasible under \(q_G\)
- are not feasible under \(q_B\) \(\Rightarrow\) constrained policy (borrow less, invest less)
Optimality Conditions (Infinite Horizon Model)

Let $\hat{\gamma} := \hat{\gamma}(k', b')$

Capital FOC

$$u'[w + q_i(w, b')b' - k'] = \beta f'(k) \left[ G(\hat{\gamma})u'[f(k) - b] + \int_{\hat{\gamma}} \gamma u'[\gamma'f(k) - k']dG(\gamma) \right]$$

Debt FOC

$$u'[w + q_i(w, b')b' - k'] = \beta \frac{1}{q_i(w, b') + \frac{\partial q_i(w, b')}{\partial b'} b'} G(\hat{\gamma})u'[f(k) - b]$$

With risk-free debt

$$q_i(w, b') + \frac{\partial q_i(w, b')}{\partial b'} b' = \frac{1}{R} \quad \Rightarrow \quad f'(k_u) = R$$
## Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share of output</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>CRRA risk aversion parameter</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Government discount factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Risk-free rate (annual)</td>
<td>$R$</td>
</tr>
<tr>
<td>Sunspot probability</td>
<td>$\pi$</td>
</tr>
<tr>
<td>Default cost distribution</td>
<td>$\gamma$</td>
</tr>
</tbody>
</table>
Spain

Surplus/GDP
Primary vs. After Interest

10y Spread

Debt/GDP

Government Gross Capital Formation/GDP

Public Employees Compensation/GDP

Taxes/GDP

GDP Growth

Aggregate Gross Capital Formation/GDP

1st Greek Rescue Package
Greek PSI
ECB's OMT
Spain
Some EZ Debt Crisis Quotes


“These urgent measures were necessary to face a serious financial crisis that has hit [...] sovereign bond markets, Italy included.”

Italian PM Mario Monti, 29/12/2011

“Our economic fundamentals do no justify such a high government bond spread.”