Asset Purchases in Noisy Financial Markets with Fiscal-Monetary Interactions*

Gaetano Gaballo
HEC Paris and CEPR

Carlo Galli
UC3M and CEPR

February 12, 2024

Abstract

We study how Asset Purchase (AP) policies affect the *real* price of defaultable nominal bonds, accounting for the effect on inflation through the central bank balance sheet. In the context of noisy financial markets where investors have position limits and private information on default probabilities, APs twist the distribution of equilibrium prices from which investors learn and effectively reduce real returns. In the absence of fiscal backing from the treasury, APs however create inflation through their effect on the real value of the central bank balance sheet. We study the social efficiency of AP policies in a stylized heterogeneous agents model, where lower bond returns and higher inflation have offsetting effects on aggregate consumption and welfare. We find that a positive but finite amount of APs optimally balances this trade-off, when we restrict to simple, uncontingent AP policies. We then show that policies that target a specific asset price can reduce interest rates while minimizing inflation pressures, even when the central bank lacks fiscal backing or has the same information as the market.

*We thank Chiara Felli, Christian Hellwig, Luigi Iovino and Vytautas Valaitis for insightful discussions of the paper. For helpful comments and suggestions, we are indebted to Marco Bassetto, Pierpaolo Benigno, Nicolas Coeurdacier, Juan Dolado, Sebastián Fanelli, Eric Mengus, Benoit Mojon, Willam Fuchs, Luigi Paciello, Evi Pappa, Josep Pijoan-Mas, Ricardo Reis, Edouard Schaal, Hernán Seoane, Victoria Vanasco, Mirko Wiederholt, and participants at various conferences and seminars. Galli gratefully acknowledges financial support from Fundación Ramón Areces; MICIN/AEI/10.13039/501100011033, with grants CEX2021-001181-M, and PID2021-122931NB-I00; and Comunidad de Madrid, with grant EPUC3M11 (V PRICIT). Gaballo acknowledges funding from the European Research Council (ERC) under the European Union's Horizon Europe research and innovation programme (Grant agreement No 101088411).

1 Introduction

The most significant change in monetary policy over the last decade is the persistent use of large-scale asset purchases (APs), also known as quantitative easing (QE). Central banks introduced this unconventional policy to overcome the zero lower bound on overnight rates and reduce long-term yields. APs also played a macroprudential role, stabilizing sovereign bond markets and mitigating large and sudden shocks such as Covid-19. In the Eurozone, they were used to fight financial fragmentation and speculative attacks. Despite their practical importance, economists continue to debate the effectiveness of QE. As Ben Bernanke famously remarked, "QE works in practice but not in theory".[1]

This paper provides a general equilibrium analysis of the social efficiency of APs, and the channels through which they work, explicitly accounting for the noisy nature of the financial markets in which it operates. We consider a setting where investors only have noisy information on the fundamental value of an asset, and take advantage of the information aggregated by the price emerging upon trade; for example, a price increase may convey the information that either other investors have positive private news about fundamentals, or that the unobservable part of the asset net supply is small. AP policies may affect such inference, even when publicly announced, in that they twist the mapping between market prices and asset supply, conditional on fundamentals.

Our contribution is twofold. First, we study this mechanism in the context of a financial market where nominal defaultable debt is traded. We characterize the impact of APs on the *real* price of bonds, i.e. accounting for the general equilibrium implications that APs have on inflation, via the balance sheet of the central bank. Second, we provide insights on the optimal AP policy relying on a stylized model of fiscal-monetary interactions where APs reduce inefficiently high interest rates, at the cost of generating socially harmful inflation.

We consider a two-period model where the players are a government, a central bank, and households. The government issues nominal, defaultable bonds in the short run (first period) to finance its stochastic spending needs, and eventually repays such debt in the long run (second period) by raising non-distortionary taxes, whose real value depends on bond interest rates as well as inflation. Repayment is a stochastic event that follows an exogenous lottery. The central bank issues money, whose long-run real value depends on that of its invested assets. The proceeds

[1] See Bernanke (2012).

of money issuance can be invested in a safe asset[2] or in government bonds.[3] The central bank may or may not receive fiscal backing of its balance sheet by the government. In particular, central bank and government interact in a regime of *monetary dominance* when fiscal transfers are set such that the value of the central bank's balance sheet is kept constant; in a regime of *fiscal dominance* instead, such transfers are absent, and a default event may affect the value of the central bank balance sheet depending on its AP policy.

The private sector consists of households holding an endowment that can be consumed in the first period, or saved and consumed in the second period. Agents are born of two types, only differing in which asset they can use in order to save: *savers* can only invest in money, whereas *investors* can invest in either bonds or the safe asset. Both types need to decide how much to consume and save in the first period before they learn any information. Once savings are set, investors receive private information on the default lottery, and decide how to allocate their portfolio between bonds and safe assets within a Bayesian trading game. In the trading game, investors face bounds on their short positions, and learn from the market-clearing bond price. Such price aggregates information about everyone's private signals, but also depends on some unobserved, stochastic supply. Investors thus face an inference problem, as they cannot tell apart whether, for example, the bond price is high because of low supply or high demand.

In the modelling of financial markets, we adopt a framework closely related to Hellwig et al. (2006), Albagli et al. (2021) and the sovereign debt application of Bassetto and Galli (2019), where the assumption of investors' risk neutrality and position limits allows considering nonlinear asset payoffs (such as that of defaultable debt). This class of models features an extensive margin mechanism, where the equilibrium price depends on the beliefs of the marginal agent, that is, the agent who is indifferent between buying government debt or investing in the alternative assets.[4]

If either heterogeneous information or position bounds are absent from the investors' portfolio allocation problem, we get a neutrality result as in Wallace (1981): there is no difference between money and bonds, the distinction between agent types is immaterial, and APs have no effect on bond prices, inflation or welfare. When instead both frictions are present, we show that APs have pervasive effects on bond prices, the information contained therein, and anything linked to such prices, specifically the central bank balance sheet, inflation, the consumption-saving decisions of

[2]This can be interpreted as a perfectly diversified portfolio, or a storage technology.

[3]Although we do not explicitly model the possibility that the central bank buys private assets, nothing prevents us from interpreting the government as the consolidated public sector.

[4]This contrasts with the key mechanism in the vast CARA-Normal noisy REE literature, where equilibrium prices depend on the risk premium priced by risk-averse investors that solve a risk-return trade-off problem. See for example Iovino and Sergeyev (2023) for an application of this framework to APs.

households, and welfare. We now briefly discuss these effects.

First, APs affect bond prices, and their mapping with fundamentals and supply shocks. When conducting APs, the central bank buys at the market price, crowding out the bond demand of a specific part of the investor distribution, that is, the least optimistic investors among those that would otherwise buy the bonds. This increases the market probability of repayment, by selecting a more optimistic investor as the marginal agent that is pricing the bond, which results in a lower bond interest rate (higher price).

Second, APs increase the precision of the information revealed by financial market prices. We show that this takes the form of a truncation in the distribution of investors' posterior beliefs on the fundamental. This result, for which we find general conditions, extends the analysis of noisy information aggregation in financial market by allowing for truncated belief distributions, being a theoretical contribution per se. This belief truncation is typically asymmetric, allowing investors to better detect default states. Intuitively, when APs are large and investors observe a high bond price, they cannot tell if that is because the government is indeed solvent, or prices are just inflated by central bank purchases. When instead APs are large and bond prices are low, investors infer that the government must be close to a default, since the price remained low even after the central bank intervened. This implies that APs render some prices fully informative of the underlying fundamental, thus eliminating all residual uncertainty in the corresponding states.

Third, APs reduce the ex ante expected value of the profits (excess returns of bonds over the safe asset) that investors make when saving and participating in the bond market. With dispersed information, agents expect to make positive profits, because they anticipate facing a call option in the bond market: if they receive a good private signal, they take on default risk and buy bonds, if not, they just save in the safe asset. This is a source of inefficiency, because it induces investors to save too much and consume too little in the first period. APs reduce these expected profits, stimulate consumption and increase welfare by reducing bond returns and revealing information in states where investors earn the most.

Fourth, in the presence of fiscal dominance, APs introduce correlation between the returns of bond and money. The long-run real value of central bank liabilities (i.e. money) depends on the real value of its investments. When the central bank invests in bonds, a default event implies a balance sheet loss that depresses the long-run real value of money and generates inflation; on the contrary, when government debt is repaid, the central bank makes a profit that generates long-run deflationary pressures. This simple result sheds light on the empirical observation that APs do not necessarily lead to inflationary pressure; on the contrary, deflation is an outcome that is perfectly consistent with the central bank investing in assets that increase the value of its liabilities. Under a simple "uncontingent" AP rule where the central bank always buys a fixed quantity of bonds, APs generate expected central bank losses and inflation. In our setting, inflation is costly for savers because it depresses the rate of return on their investment, money, below its efficient level. Hence, a trade-off emerges: on the one hand, APs reduce inefficiently high return for investors and increase their consumption; on the other hand, APs increase inflation, make the rate of return on money inefficiently low, and reduce savers' consumption. We show that the optimal uncontingent AP policy is to buy a positive but finite amount of bonds, trading off the welfare gains for investors with the losses for savers.

Finally, we study a more sophisticated yet tractable class of AP policies that target a specific bond interest rate. These policies are implemented as limit orders by the central bank to buy up to a certain quantity of bonds if the price is weakly smaller than a given target. Importantly, this class of polices does not require the central bank to know the fundamental shocks in the economy, or to have information that is superior to that of investors. We show that price-targeting policies have two important features: first, they are "beliefs-neutral", in the sense that they do not distort the information contained in the price, and actually correct a wedge that derives from the presence of information frictions; second, they are "budget-neutral", because we set a price target such that APs result in zero expected profits or losses for the central bank. This implies that, with this class of policies, the central bank can reduce interest rates and increase investors' consumption, while minimizing the drawback of creating costly inflation for savers.

Related Literature. Since Wallace (1981), the irrelevance of open market operations has been a benchmark theoretical result in rational expectation macroeconomic models. It states that, taking fiscal policy as given, any purchase of assets by public authorities is allocation-neutral insofar as taxes adjust to offset any gain or loss in public budgets. Thus, the composition of public liabilities does not matter, similarly to what Modigliani and Miller (1958) show for corporate liabilities.

Wallace's irrelevance result crucially obtains under complete information and frictionless financial markets. A literature questioning the complete information assumption focused on the role of APs to serve as a signal about uncertain central banks' objectives and fundamentals (see Mussa (1981)) or as a commitment device to future accommodative stance (Jeanne and Svensson (2007), Christensen and Rudebusch (2012) and Bhattarai et al. (2022)). Recent work by Iovino and Sergeyev (2023) focuses on the lack of rational expectations as a source of non-neutrality of APs in an otherwise frictionless model. A larger stream of literature has emphasized the importance of market segmentation for the workings of AP policies, in the vein of seminal papers like

Cúrdia and Woodford (2011) and Gertler and Karadi (2015).[5] Some papers have emphasized the role of asset purchases in incomplete markets economies with structurally heterogeneous agents in alleviating a lack of risk sharing or insurance on the side of firms or households.[6]

To the best of our knowledge, this is the first paper showing the role of dispersed information in economies where structurally homogeneous investors take bounded positions. In particular, we show that absent one of these two frictions – private uncertainty or bounded asset demand – the neutrality benchmark obtains. In our model, positions bounds prevent private demand to perfectly offset the heterogeneous crowding-out effect of APs, which has effects on the asset price and the information private agents extract from it.

Such absence of perfect private arbitrage echoes the assumption of market segmentation that is common in the finance literature on asset purchases. Market segmentation is essential for APs to induce "portfolio rebalancing" effects, i.e. a relative price change across asset classes and maturities. These effects have been measured since the great recession of 2008-2009 in a flourishing empirical literature.[7] Some works support the view that asset purchases have mostly a "local" effect limited to the specific market targeted by the program.[8] Others have identified sizeable "global" portfolio rebalancing effects that pervade financial markets beyond those that are targeted directly by the program.[9]

On the theoretical front, the literature has developed models to account for the "local" vs. "global" effects of asset purchases on financial markets, building on the seminal paper by Vayanos and Vila (2021) (see for example Hamilton and Wu (2012), Greenwood and Vayanos (2014), King (2013) and King (2019)). All these papers focus on the financial market impact of APs, abstracting from their general equilibrium implications on inflation and macroeconomic risks. An applied macro literature found robust evidence for expansionary general equilibrium effects of asset purchases.[10]

[5]See for example Chen et al. (2012), Del Negro et al. (2017), Wen (2014), Campbell et al. (2012), Harrison (2017) and Sims and Wu (2021).

[6]See for example Gornemann et al. (2016), Auclert (2019), Luetticke (2018), Ravn and Sterk (2021), Kaplan et al. (2018), Debortoli and Galí (2017), Hagedorn et al. (2019) and Cui and Sterk (2021).

[7]See Gagnon et al. (2011) for the US; Joyce et al. (2012) and Breedon et al. (2012) for the UK, and more recently Koijen et al. (2021) and Altavilla et al. (2021) for the Eurozone, among others.

[8]See Krishnamurthy and Vissing-Jorgensen (2011), D'Amico and King (2013), and McLaren et al. (2014) on Fed LSAPs; Eser and Schwaab (2016) for the ECB Securities Markets Programme (SMP); Altavilla et al. (2016) for the ECB Outright Monetary Transactions (OMT); Krishnamurthy et al. (2017), Koijen et al. (2017) and Arrata et al. (2020) for various ECB AP programs.

[9]E.g. Cahill et al. (2013), Li and Wei (2013), Gilchrist et al. (2015) and Rogers et al. (2018).

[10]See Bhattarai and Neely (2016) and Kim et al. (2020) for a survey of the literature.

2 Model

There are two periods $t \in \{1,2\}$. In the first period, a continuum of agents chooses how much endowment to allocate to current consumption rather than savings in a asset. Agents are of two types, each sizing mass one, that differ in the saving assets they have access to. *Savers* can only save in money, *Investors* can choose between a safe return asset and a defaultable nominal bond issued by the government. In the second period, once the consumption-saving choice had been made, investors receive a private signals on the likelihood of default, see the market price of bonds and decide how to compose their portfolio. At the end of the second period, non-defaulted bonds and money are reimbursed, consumers pay lump-sum taxes and lastly consume.

The public sector consists of a government and a central bank, both implementing rules or the conduct of fiscal and monetary policies. In the first period, the government issues bonds to finance consumption and the central bank issues money and implements APs of government bonds. In the second period, government default may occur or not depending on the outcome of an exogenous lottery, the government raises lump-sum taxes to repay the non-defaulted debt and operates transfers with the central bank.

2.1 Monetary-Fiscal interactions

The government's budget. The government issues a quantity of defaultable nominal bonds to finance the realization of stochastic public consumption needs in the first period, $\widetilde{S}$, which follows a Uniform$[0,1]$ distribution. A unit of bonds is a promise by the government to pay R units of money in the second period, in exchange for one unit of money in the first period. We assume the price of consumption in the first period as a numeraire, $P_1 = 1$, so that the nominal amount of bonds issued by the government is equal to its real needs $\widetilde{S}$. The real return of bonds depends on the occurrence of default and inflation in the second period. Default is an exogenous event occurring stochastically according to the following lottery:

$$\theta = \begin{cases} \theta_H = 1 & \text{with probability } \; q, \\ \theta_L \in (0,1) & \text{with probability } \; 1-q. \end{cases} \tag{1}$$

where θ denotes the fraction of debt effectively repaid by the government. Inflation $\Pi := P_2/P_1$ occurs when there is variation in the price of consumption between the first and the second period. Finally, the government raises real resources by collecting lump-sum taxes T. The budget set of the government in the second period is given by

$$T = \frac{R\,\theta}{\Pi}\,\widetilde{S} + \tau \tag{2}$$

where τ represents eventual transfers from the government to the central bank. Throughout the paper we assume the two realizations $(\theta, \widetilde{S})$ are unobservable to households. Effectively, θ determines whether a default occurs or not, so it is natural to assume it to an unobservable fundamental component of the assets value. Bond supply $\widetilde{S}$ should be broadly interpreted as any type of disturbance that may move the aggregate asset supply irrespective of fundamental values, so to create confusion on the sources of price fluctuation; this is, for example, achieved thought the fiction of noisy traders in a large stream of literature in finance.

The central bank's budget. The central bank has an initial endowment e_{cb} and issues a stock of money m to buy a nominal quantity of government bonds b_{cb} or invest in a safe asset s_{cb}, so that

$$e_{cb} + m = b_{cb} + s_{cb} \tag{3}$$

is the budget constraint of the central bank in the first period. In the second period, the central bank collects bond and safe asset revenues, reimburses money and transfers the endowment to the government.[11] The budget constraint of the central bank in the second period is therefore:

$$\frac{R\,\theta}{\Pi}\,b_{cb} + s_{cb} + \tau = \frac{m}{\Pi}. \tag{4}$$

where, without loss of generality, we normalize the real return of the safe asset to one. The transfer τ is critical for the determination of monetary fiscal interactions. We consider the following generic rule for transfers:

$$\tau = \left(1 - \frac{R\,\theta}{\Pi}\right)\kappa\,b_{cb} - e_{cb}, \tag{5}$$

where $\kappa \in [0, 1]$ measures the degree of fiscal backing by the government: $\kappa = 0$ entails no backing, whereas $\kappa = 1$ is perfect fiscal backing as any loss or profit is transferred entirely to the

[11] The inclusion of an initial endowment of the central bank and the assumption that money is reimbursed at the end of the second period are artefacts of the finite nature of time considered here. Appendix XXX shows that, in an infinite horizon OLG economy, the central bank's endowment is replaced by proceeds from previous central bank savings, and the old households sell their money balances to the young.

fiscal authority. Hence, the rate of return on money obtains as

$$\frac{1}{\Pi} = \left((1-\kappa)\,\frac{R\,\theta}{\Pi} + \kappa \right) \alpha + (1-\alpha), \tag{6}$$

where $\alpha := \frac{b_{cb}}{m}$ represents the fraction of money stock invested in asset purchases. We will refer to $\hat{\alpha} := (1-\kappa)\alpha$ as the degree of fiscal dominance: when $\hat{\alpha} = 0$, inflation is equal to 1 irrespective of the default realization. The rate of the return on money $1/\Pi$ is a weighted average, with weight $\hat{\alpha}$, of the real return on bonds $R\,\theta/\Pi$ and on the one on safe asset 1; as $\hat{\alpha}$ increases, the correlation between the real return on money and bonds increases.

In particular, we denote the regime where $\kappa = 1$ as *monetary dominance*. In this regime, the budget constraint of the central bank holds with $\Pi = 1$ for any values of (θ, R, m, b_{cb}). This implies that the value of money is stable irrespective of fiscal variables, since losses or gains from central bank APs are perfectly offset by transfers to or from the fiscal authority. When instead $\kappa = 0$, we will say that taxes are determined under the *fiscal dominance* regime: transfers to the central bank do not vary with profits (or losses) from APs, and inflation Π must adjust for (4) to hold. The value of money must thus fluctuate with profits and losses from APs, inflating or deflating the value of central bank liabilities (money), to balance the budget. Given $\kappa = 0$, the higher the α, the higher the share of the central bank assets that are invested in bonds, the closer the real return of money is to the one on bonds.

AP policy rule. We consider a class of asset purchase policies that we denote as "price-targeting" policies. These policies are characterised by an interval of feasible purchased quantities $[0, \bar{b}_{cb}]$ and a target equilibrium interest rate R_n. When bidding in the bond market, the central bank submits, simultaneously to investors, a limit order to buy up to $\bar{b}_{cb}$ at an interest rate larger than or equal to the target R_n. The rule can be represented formally by the following equilibrium requirement:

$$b_{cb} = \underset{b_{cb} \in [0, \bar{b}_{cb}]}{\operatorname{argmin}} \, |R(b_{cb}) - R_n| \tag{7}$$

implying that, in equilibrium, APs $b_{cb} \in [0, \bar{b}_{cb}]$ are such that the resulting market interest rate is the closest possible to target R_n. The notation $R(b_{cb})$ is adopted to remark that, being a non-atomistic agent, the central bank purchases impact on the market price, i.e. lowering bond returns. As we will see, since $R'(b_{cb}) < 0$, APs are maximal with $R(\bar{b}_{cb}) > R_n$ and zero when $R(0) < R_n$. Moreover, note that by fixing the target to the lowest possible interest rate, i.e. $R_n = 1/\theta_H$, APs are maximal at any price. This case entails an *uncontingent* APs policy: the central bank buys the same amount $b_{cb} = \bar{b}_{cb}$ at any price. It is important to remark that, to implement price-targeting AP polices, the central bank does not need to know neither gross bond supply $\widetilde{S}$, nor the fundamental θ. The limit order that the central bank submits must only specify a price target and an upper bound for quantities.

2.2 The private sector

A household, denoted by j, has a quasi-linear utility function $U_j \equiv u(c_{j,1}) + c_{j,2}$, where utility is increasing and concave in first-period consumption $c_{j,1}$, and linear in second-period consumption $c_{j,2}$. Each household has a productive endowment $e_j = 1+c^*$. At the beginning of the first period, agents decide how much of the endowment to allocate to consumption $c_{j,1}$ and savings a_j. At the beginning of the second period, agents receive payoff-relevant information Ω_i, and make a portfolio choice b_j, which we describe in detail in the following paragraph. In the second period, agents receive the return from their savings, which depends on their previous portfolio choice $\mathcal{R}_j(b_j)$, pay lump-sum taxes $T/2$ and consume $c_{j,2}$. Formally, household j solves the following consumption-savings problem:

$$\max_{a_j \in [0,\, e_j]} \mathbb{E}\left[u(c_{j,1}) + c_{j,2}\right], \tag{8}$$

subject to

$$c_{j,1} = e - a_j, \tag{9}$$
$$c_{j,2} = a_j \max_{b_j} \mathbb{E}\left[\mathcal{R}_j(b_j)\,|\,\Omega_j\right] - \frac{T}{2}. \tag{10}$$

Let us now describe the portfolio choice. A structural heterogeneity exists in that households differ in the type of financial asset they have access to. We distinguish between two types, each of unitary mass.

We refer to the first type as *savers*, denoted by $j = s \in [0,1]$. These agents do not face a portfolio problem, and can only save in the form of money, i.e. b_s is indeterminate and in equilibrium $a_s = m$. Their savings thus yield a real rate of return equal to the inverse of the gross inflation rate

$$\mathcal{R}_s = \frac{1}{\Pi}. \tag{11}$$

Moreover, savers' information set Ω_s does not contain additional information other than the prior distribution of θ and $\widetilde{S}$. Savers generate demand for money and, importantly, fluctuations in expected inflation have consequences for their saving-consumption choice and, therefore, for aggregate consumption.

The second type of household, denoted by $j = i \in [0,1]$, are *investors*, who do have access to a financial market where they face a portfolio problem. In the financial market, investors must allocate their savings a_i into two types of assets: nominal defaultable government debt, and a safe asset (e.g., a fully diversified portfolio) that has a real, risk-free rate of return of unity, and is available in infinitely elastic supply. We assume that investors can sell bonds short up to a fraction $\underline{b}$ of their savings. Short-selling of the safe asset is instead not allowed. Each investor i thus chooses what fraction $b_i \in [-\underline{b}, 1]$ of its savings to invest in government bonds, so that her per-unit rate of return is

$$\mathcal{R}_i(b_i) := b_i \frac{R\,\theta}{\Pi} + (1 - b_i)\,, \tag{12}$$

and is thus determined for a fraction b_i by the real return on the bond, and for the rest by the return on the safe asset. Importantly, this setting introduces an endogenous bound to the asset position of households, since they cannot invest more than the savings they decided, and short-sell beyond an exogenous limit $-\underline{b}$. Because of risk neutrality in second period consumption, bounds will be binding, except for a knife-hedge case, preventing full arbitrage in the financial market. The lack of complete arbitrage due to position bounds is a key assumption for APs to have an effect. We show in Appendix A that, in a noisy financial market with preference-induced finite asset position (e.g. risk aversion), observable APs that are fully backed by lump sum taxation will perfectly crowd out private demand, leaving the equilibrium price unaffected. This is the well-known neutrality result of Wallace (1981).

2.3 Market Clearing and Equilibrium

Market clearing The equilibrium prices in the economy Π and R are such that money, bond and good markets clear, that is

$$\text{money:} \quad \int_0^1 a_s \mathrm{d}s = m, \tag{13}$$

$$\text{bonds:} \quad \int_0^1 a_i\, b_i\, \mathrm{d}i + b_{cb} = \widetilde{S}, \tag{14}$$

$$\text{goods:} \quad \int_0^2 c_{j,1}\, \mathrm{d}j + \int_0^2 c_{j,2}\, \mathrm{d}j + \widetilde{S} = 2\,e. \tag{15}$$

In particular, a market return R is such that the orders submitted by investors and central bank clear bond government's supply. Hence, the equilibrium interest rate R will generally depend on the repayment state θ, gross supply $\widetilde{S}$, the asset purchases b_{cb} and position bounds $\underline{b}, a_i$.

Equilibrium We are now ready to give a general definition of an equilibrium. We divide our definition in two parts. First, a notion of competitive equilibrium that generically depends on information sets of agents. Second, a rational expectation equilibrium, in which agents learn from all endogenous aggregate variables being the price R and the APs b_{cb}, by conditioning on the correct equilibrium distribution of these objects.

Definition 1. *For given differentiable and concave utility function $u(\cdot)$, households information sets $\{\Omega_j\}_{j\in[0,2]}$, households' and central bank's endowments $\{e_{cb}, \{e_j\}_{j\in[0,2]}\}$, a degree of fiscal backing κ, a central bank's APs range $[0, \bar{b}_{cb}]$ with price target R_n, a short selling bound $\underline{b}$, and a lottery (1) with default probability $1-q$, a competitive equilibrium is characterised by a price function*

$$R(\theta, \widetilde{S}, b_{cb}) : \{\theta_H, \theta_L\} \times [0,1] \to \mathbb{R}^+, \tag{16}$$

a collection of individual market policies $\{\{a_j\}_{j\in[0,2]}, \{b_i\}_{i\in[0,1]}\}$ and an AP-policy b_{cb}, such that, at any state $(\theta, \widetilde{S})$:

i) the budget sets of the government and the central banks (2)-(3)-(4) hold, transfer occur according to (5), and inflation obtains as (6);

ii) the APs rule (7) is satisfied;

iii) the consumption and portfolio problems of households (8)-(9)-(10)-(11)-(12) are solved;

iv) the bond, money and goods market clear according to (13)-(14)(15);

(v) the consumption allocations $\{c_{1,j}, c_{2,j}\}_{j\in[0,2]}$ are pinned down.

A Rational Expectation Equilibrium (REE) is one in which investors learn about θ from the market price and central banks's APs, that is $(R, b_{cb}) \in \Omega_i$, conditional on the price equilibrium distribution (16) and the target rule (7), respectively.

Our definition emphasises the return on the financial market as key object characterising the equilibrium. Section 3 will be devoted to the building of this mapping, clarifying the differences emerging from the requirement that investors have rational expectations. Before diving into it, in the rest of this section we will clarify under which conditions the market can generate the first best allocation and which externalities are responsible for eventual departures.

2.4 Market vs Planner

Individual market policies. We clarify here how individual market policies a_j and b_i as functions of the rate of return prevaling in the financial market where agent j has access to. First, let us focus on the consumption-saving decision. The amount of savings a_j that agent j decides to hold is given by the following Euler equation

$$u'(c_{j,1}) = \mathbb{E}\left[\max_{b_j} \mathbb{E}\left[\mathcal{R}_j(b_j)\,|\,\Omega_j\right]\right], \tag{17}$$

i.e., the marginal utility of first-period consumption must equate to the expected real return on saving. Without loss of generality, we fix $u'(c^*) = 1$, so that $a_j = 1$ is the amount of savings that equates marginal utility to the inverse of the discount factor, assumed equal to one for simplicity. The Euler equation (17) entails the traditional intertemporal substitution motive at the core of workhorse models of the business cycle: an expected real return above (resp. below) the natural level (1 in our case) induces a negative (resp. a positive) gap of current consumption $c_{1,j}$ with the natural level c^*. According to (11), the real return for savers is equal to the inverse of inflation, so that, $\mathbb{E}[1/\Pi] < 1$ implies $c_{1,s} > c^*$. For investors the same logic applies, but according to (12), the return that they get on the financial market is a function of their portfolio choice, that they anticipate when deciding on their savings.

Let us then look at the portfolio choice of investors. Because of risk neutrality, agent i with information set Ω_i chooses

$$b_i = \begin{cases} 1 & \text{if and only if} \quad \mathbb{E}\left[\dfrac{R\,\theta}{\Pi}\,|\,\Omega_i\right] > 1, \\ \text{indeterminate} & \text{if and only if} \quad \mathbb{E}\left[\dfrac{R\,\theta}{\Pi}\,|\,\Omega_i\right] = 1, \\ -\underline{b} & \text{if and only if} \quad \mathbb{E}\left[\dfrac{R\,\theta}{\Pi}\,|\,\Omega_i\right] < 1, \end{cases} \tag{18}$$

where in case of indeterminacy the investor is ready to buy (or short-sell) any quantity within the bounds $[-\underline{b}\,a_i\,,\,a_i]$. Investors submit demand schedules contingent on the market clearing interest rate R (i.e., the inverse of the bond price), and the available information summarised by Ω_i, to be specified later. It immediately follows that $\mathbb{E}\left[\mathcal{R}_i(b_i)\right] \geq 1$, that is, since investors have the outside option of buying the safe assets, their expected return is never lower than the safe rate. In particular, as we will see in detail later, the possibility of saving returns above the natural rate depends on the presence of exogenous bounds to asset positions, that by limiting the purchases of relatively more optimistic investors, generates a premium to be paid by the government for the market to clear.

Welfare. To better understand how frictions in the financial market may generate suboptimal outcomes, let us work out the first best allocation in this economy. Firstly, let us define social welfare as the sum of utilities of agents and government, i.e. is equal to total utility from consumption. Using (2)-(14), aggregate welfare is given by

$$\mathcal{W} := \int_0^2 U_j \,\mathrm{d}j + \widetilde{S} = u(e_s - m) + u(e_i - a_i) + m + a_i + e_{cb}. \tag{19}$$

One can easily note that only consumption-saving choices matter for welfare, whereas portfolio choices by investors in the financial market do not. In fact, the market for bonds accounts only for a liquidity friction on the side of the government and does not generate any new production. It is immediate to realise that the first best allocation, entailing maximum welfare $\mathcal{W}^* := 2\,u(c^*) + 2 + e_{cb}$, is the one characterized by $a_s = a_i = 1$, that is, when the marginal utility of first-period consumption for any household is equal to the natural rate.

Market implementation of the first best. Under which condition can the market entail the first best allocation? The first best allocation can be achieved by the market when the expected returns on savings are equal to the natural rate, that is, $\mathbb{E}[\mathcal{R}_i(b_i)] = \mathbb{E}[1/\Pi] = 1$. This occurs when investors share the same information as the following propositions states.

Proposition 1. *Suppose investors have homogeneous information, i.e.* $\Omega_i = \bar{\Omega}$ *for each* i*, then the market achieve then money and bonds yields the same real return* $\mathcal{R}_i(b_i) = \Pi = 1$ *so that the first best allocation* $a_s = a_i = 1$ *is achieved.*

Proof. See Proof 1 in the appendix. ■

The proposition states that investors' homogenous information makes the distinction between money and bonds and the segmentation of the financial market irrelevant as the market is still able to implement the first best. In this sense, our key friction is the heterogeneity of information across investors. When investors have disparate opinions on the likelihood of full repayment, the market solution, may not necessarily implement the first best allocation, for two key reasons. First, investors do not internalize that taxes in the second period vary with the market price, which ultimately depends on investors' actions as a whole. Second, given taxes, there are strictly positive gains to be done in the financial market exploiting private information since the presence of bounds to asset positions prevents investors' full arbitrage.

3 Equilibrium return with dispersed beliefs

In this section we derive the price mapping (16) that characterises an equilibrium in presence of heterogeneous information. We will first derive a mapping from a *given* distribution of investors' posterior beliefs to the market clearing return. Then we will discuss the information set and how posterior beliefs are formed in a REE. We will finally discuss the properties of the REE price equilibrium mapping.

3.1 Equilibrium Price Given Marginal Investor Beliefs

In this subsection, we derive a generic characterisation of the equilibrium bond price in the financial market, as a function of the beliefs of the *marginal investor*, which represent a sufficient statistic for a given distribution of investors' posterior beliefs. This is useful because the mapping between the marginal investor's beliefs and the equilibrium in both the financial market and the macroeconomic model are independent of the way in which such beliefs are formed.

We now focus on the investors' problem in the financial market and the equilibrium therein. Investors' total savings (and upper bound for long positions in bonds) a_i are decided before entering this stage of the first period, when their information set is homogeneous, which implies that they will be the same for all agents. To reflect this fact and lighten up notation, we thus omit subscript i and denote total savings with a for the rest of this section.

Let us denote the repayment probability held by investor i by $p_i := Prob(\theta = \theta_H \,|\, \Omega_i) \in [0, 1]$, which is distributed according to a generic distribution G. It follows that the expected value of the fundamental θ for an investor having p_i is given by $\mathbb{E}[\theta \,|\, \Omega_i] = \theta_L + p_i(\theta_H - \theta_L)$, which is strictly increasing in p_i. Using equation (6) and provided that $R\,\theta < 1/\hat{\alpha}$, we get that the real bond payoff is

$$\frac{\theta}{\Pi} = \frac{1 - \hat{\alpha}}{\frac{1}{\theta} - \hat{\alpha} R}, \tag{20}$$

where $\hat{\alpha} = (1 - \kappa)\alpha$, so that for $\kappa = 1$ or $\alpha = 0$ it is easy to verify that $\Pi = 1$. It is also easy to check that monotonicity in beliefs on repayment maps into monotonicity in beliefs about ex-post real returns for a given R, that is:

$$p_i \geq p_j \quad \Leftrightarrow \quad \mathbb{E}\left[\theta \,|\, \Omega_i\right] \geq \mathbb{E}\left[\theta \,|\, \Omega_i\right] \quad \Leftrightarrow \quad R\,\mathbb{E}\left[\frac{\theta}{\Pi} \,|\, \Omega_i\right] \geq R\,\mathbb{E}\left[\frac{\theta}{\Pi} \,|\, \Omega_i\right], \tag{21}$$

for any degree of fiscal dominance $\hat{\alpha} \in [0,1)$ and pair of agents $(i,j) \in [0,1]^2$. We introduce then the following definition.

Definition 2 (Marginal agent and market clearing price). *For a given net supply, $\widetilde{S} - b_{cb}$, the marginal agent $m \in [0,1]$ is the investor who holds posterior beliefs $p_m \in [0,1]$ such that the market clears with*

$$a\,(1 - G(p_m)) - a\,\underline{b}\,G(p_m) = \widetilde{S} - b_{cb} \tag{22}$$

where $G(p_m)$ is the mass of investors who are more pessimistic than the marginal investor, and short-sell bonds on the market.

The marginal investor is the equilibrium sufficient statistics of the distribution of investors beliefs. It yields the equilibrium restriction that defines the equilibrium return in the financial market. Such restriction is given by (18) where, by construction, b_m is the value indeterminate at the equilibrium. From that relation, the market clearing price R is determined by

$$R\,\mathbb{E}\left[\frac{\theta}{\Pi}\,\Big|\,\Omega_m\right] \;=\; R\left[p_m\,\theta_H \frac{1-\hat{\alpha}}{1-\hat{\alpha}R\theta_H} + (1-p_m)\,\theta_L \frac{1-\hat{\alpha}}{1-\hat{\alpha}R\theta_L}\right] \;=\; 1, \tag{23}$$

so that R is the price that makes the marginal investor indifferent between buying or short selling. Line (23) defines the equilibrium fix point equation, which then yields the equilibrium return R as a function of the posterior of the marginal investor p_m and the characteristics of the default lottery. This is stated in the following proposition.

Proposition 2. *Provided $R\,\theta < 1/\alpha$ for any $\theta \in \{\theta_L, \theta_H\}$, then, for a given belief of the marginal agent p_m, the equilibrium interest rate $R(p_m) : [0,1] \to [1/\theta_H, 1/\theta_L]$ is given by*

$$R\,(p_m) = \frac{(1-\hat{\alpha})\mathbb{E}[\theta \,|\, \Omega_m] + (\theta_H + \theta_L)\hat{\alpha} - \sqrt{((1-\hat{\alpha})\mathbb{E}[\theta \,|\, \Omega_m] + \hat{\alpha}(\theta_H + \theta_L))^2 - 4\hat{\alpha}\theta_H\theta_L}}{2\hat{\alpha}\theta_H\theta_L}. \tag{24}$$

The equilibrium price function $R(p_m)$ has the following properties:

i. it is monotonically decreasing in the posterior of the marginal agent p_m with:

$$\lim_{p_m \to 0} R(p_m) = \frac{1}{\theta_L} \;>\; \lim_{p_m \to 1} R(p_m) = \frac{1}{\theta_H}, \quad \textit{with} \quad \frac{\partial R(p_m)}{\partial p_m} < 0;$$

ii. it is monotonically decreasing in the degree of fiscal dominance $\hat{\alpha}$ with:

$$\lim_{\hat{\alpha}\to 0} R(p_m) = \frac{1}{\mathbb{E}[\theta|\Omega_m]}, \quad \lim_{\hat{\alpha}\to 0} \frac{\partial R(p_m)}{\partial \hat{\alpha}} = 0, \quad \frac{\partial R(p_m)}{\partial \hat{\alpha}}\Big|_{\alpha\neq 0} < 0.$$

Proof. Postponed to Proof 2 in the appendix. ■

The proposition characterises the one-to-one mapping $R(p_m) : [0,1] \to [1/\theta_H, 1/\theta_L]$ between the beliefs of the marginal agent and the equilibrium price or interest rate. The first part states that the equilibrium nominal return is equal to its minimum $1/\theta_H$ when the marginal agent believes repayment occurs with certainty, and it increases as the marginal agent considers default more likely, until it reaches its maximum $1/\theta_L$, where the marginal agent believes default occurs with certainty. Intuitively, a higher probability of repayment by the marginal agent maps into a lower equilibrium return because a smaller remuneration is sufficient to clear the market when the distribution of beliefs in the population is more optimistic.

The second part of the proposition states that a higher degree of fiscal dominance $\hat{\alpha}$ decreases the equilibrium *nominal* interest rate. To see why, recall first that fiscal dominance makes inflation comove with the fundamental: in repayment states, central bank profits from APs generate deflation, which increases real bond returns; in default states instead, central bank losses generate inflation, which reduces real bond returns. In the appendix we show that the former effect is always stronger than the latter, so R increases with $\hat{\alpha}$ for any marginal agent belief p_m.

3.2 Information

3.2.1 Exogenous Information: APs and Private Signals

Uncontingent AP rule. Through the whole section we will consider the simplest case for the AP rule (7) by fixing the price target to the lowest return, $R_n = 1/\theta_H$, so that APs will be a fixed amount $\bar{b}_{cb}$ at any $R > 1/\theta_H$.

For the remainder of Section 3, we focus on the simplest case that the central bank targets the highest possible price θ_H, that is, it intervenes by buying the maximum at any price. This result in an uncontingent quantity policy, allowing to study the workings of APs in the simplest case. In Section 5, we instead show that the optimal price-targeting policy must be one that targets a particular price target.

As anticipated in Subsection 2.1, here we assume that the central bank follows a price-targeting policy where it submits a limit order to buy up to $\overline{b}_{cb}$ units of the bond at an interest rate larger than or equal to $R_n = 1/\theta_H$. Since this target is the lowest possible interest rate prevailing in the market, this policy is equivalent to assuming that the central bank is always intervening in the market. The amount of bonds it buys must however be such that net supply is non-negative, because investors demand cannot be negative. It follows that if total bond supply (by government and short-sellers) is below $\overline{b}_{cb}$, the central bank buys all the supply.[12] If instead total supply is above $\overline{b}_{cb}$, then the central bank buys $\overline{b}_{cb}$ at the prevailing market interest rate.

The quantity of central bank purchases that implements its policy is given by

$$b_{cb} = \min\left\{\widetilde{S} + a\,\underline{b}, \overline{b}_{cb}\right\} \qquad \text{and} \qquad \overline{b}_{cb} \in [0, 1 + a\,\underline{b}] \tag{25}$$

for any $(\theta, \widetilde{S})$. With a little abuse of language, we refer to this policy as *uncontingent* because the central bank is always intervening, and even the quantity of bonds it buys only depends on $\widetilde{S}$ for feasibility reasons. As we will discuss later, this is a conservative assumption in evaluating central bank losses. We denote with $P_0 := \overline{b}_{cb} - a\,\underline{b}$ the probability that the central bank purchases the whole bond supply, that is, no supply is available to buyers in the market. We refer to this scenario by saying that the market is *passive*. This is a slight abuse of notation, because when $\underline{b} > 0$ investors will all be selling the bonds short in this circumstance. It is only when $\underline{b} = 0$ that the market is truly passive, in the sense that no investor is taking any short or long position in bonds. It follows that with probability $1 - P_0$ there is a non-zero mass of investors buying bonds in the market, in which case we say the market is *active*.

Private information on default. In period $t = 1$, bond investors do not observe the realization of θ, but they may receive information about it: we denote the information set of investor i in stage 1 with Ω_i. We assume that each agent has a private noisy signal on θ given by

$$x_i = \theta + \sigma_x\,\xi_i, \tag{26}$$

[12] In this case, the central bank will buy at the full information interest rate $R = 1/\theta$. The reason behind this will be clearer later on in the analysis, and is explained in detail in Section 4.2 where we discuss central bank profits and losses from APs.

where $\xi_i \sim N(0,1)$ for each i are mutually orthogonal white noise shocks. We denote the unconditional distribution of x_i with $\mathcal{N}_x$.[13]

3.2.2 Endogenous Information: the Price Signal

The distribution of the marginal investor. We can now derive explicitly how the mass of investors taking long and short bond positions is determined in equation (22). First, whenever the market price does not fully reveal the value of θ, posterior beliefs are increasing in the private signal x_i in the sense of first-order stochastic dominance. Second, investors' expected payoff are an increasing function of beliefs, as shown in (21). This implies that agents follow monotone threshold strategies, and we can rewrite equation (18) as

$$b_i(x_m) = \begin{cases} 1 & \text{if} \quad x_i \geq x_m, \\ -\underline{b} & \text{if} \quad x_i < x_m \end{cases} \tag{27}$$

where $b_i(x_m)$ is the bond position taken by agent i when the private signal threshold is x_m, which is endogenous to the equilibrium. We assume that a law of large numbers across investors applies as in Judd (1985): for a given value of the fundamental, the mass of investors buying bonds is given by the share of agents with a private signal larger than x_m, that is

$$1 - G(p_m) = \text{Prob}(x_i \geq x_m \,|\, \theta) = \Phi\left(\frac{\theta - x_m}{\sigma_x}\right),$$

where Φ denotes the standard normal cumulative distribution function. The cutoff private signal x_m identifies the marginal agent on the market, i.e., the investor whose private signal is such that she is indifferent between buying the bonds or investing in the safe asset. Rearranging equation (22) we can express the private signal of the marginal agent as

$$x_m(\theta, S) = \theta + \sigma_x \Phi^{-1}(1 - S). \tag{28}$$

[13] The unconditional distribution of x_i is given by

$$f_{x_i}(x) = \sum_{j \in \{L,H\}} q_j f_{x_i}(x|\theta_j) = \frac{1}{\sigma_x}\left[q\,\phi\left(\frac{\theta_H - x}{\sigma_x}\right) + (1-q)\,\phi\left(\frac{\theta_L - x}{\sigma_x}\right)\right]$$

where $q_H = q$ and $q_L = 1 - q$.

where

$$S(\widetilde{S}, \mathbf{b}) := \frac{\widetilde{S} + a\,\underline{b} - b_{cb}}{a(1+\underline{b})} \sim \text{Uniform}\,[S_{min},\ S_{max}] \tag{29}$$

defines *net bond supply per individual exposure*, or simply *net supply*, as the supply available to each buyer, in units of individual exposure $\Delta := (a(1+\underline{b}))^{-1}$, given position bounds and APs. We summarise with $\mathbf{b} := \{a, b_{cb}\}$ the level of investors' savings and of central bank APs.

Equation (28) states that the marginal agent's private signal x_m must be equal, in equilibrium, to a function that is linear in the fundamental shock θ, and nonlinear in the gross supply shock $\widetilde{S}$, the central bank's AP policy, and the bond position bounds. Henceforth, we will focus on equilibria where x_m and R convey the same information, in which case conditioning beliefs on the marginal agent signal x_m is equivalent to conditioning them on the endogenous price R. We thus refer to x_m as the price, or market, signal, which is a public signal that is endogenous to the equilibrium.

We use $\mathcal{M}(\mathbf{b})$ to denote the unconditional distribution of x_m. Note that this distribution is not necessarily the same as the private signal distribution $\mathcal{N}_x$, because of market clearing restrictions. That is, the support of net supply S depends on the central bank AP policy, as well as the bond position bounds. In particular, $\mathcal{M}(\mathbf{b}) = \mathcal{N}_x$ if and only if $[-\underline{b}, a] = [0, 1]$ and $b_{cb} = 0$ for any $(\theta, \widetilde{S})$.[14] More generally, the support of the marginal investor distribution ranges from $S_{min} = (a\,\underline{b} - b_{cb})\,\Delta \geq 0$ to $S_{max} = (1 + a\,\underline{b} - b_{cb})\,\Delta \leq 1$. This implies that the support of x_m conditional on θ may differ from the support of x_i, and ranges from $x_{min} := x_m(\theta_L, S_{max})$ to $x_{max} := x_m(\theta_H, S_{min})$.

It is important to note that, since the support of x_m depends on θ and may have finite bounds, there may exist an upper interval of price signals $[x_+, x_{max}]$ that realise only if $\theta = \theta_H$, and a lower interval $[x_{min}, x_+]$ that realise only if $\theta = \theta_L$. We define S_- and S_+ as the values of net supply that correspond to (x_-, x_+):

$$\begin{aligned} S_- &: x_+ := x_m(\theta_L, S_{min}) = x_m(\theta_H, S_-), \\ S_+ &: x_- := x_m(\theta_H, S_{max}) = x_m(\theta_L, S_+). \end{aligned} \tag{30}$$

In practice, S_- (resp. S_+) is the value of net supply at which, when θ is high (resp. low), the marginal investor receives the same private signal that the most (resp. least) optimistic marginal investor would receive when θ is instead low (resp. high). This means that observing any $x_m \in (x_+, x_{max}] \cup [x_{min}, x_-)$ is revealing of the underlying value of θ. On the contrary,

[14] This can be shown using the fact $\Phi(S) \sim N(0, 1)$ if $S \sim \text{Uniform}[0, 1]$.

observing any $x_m \in [x_+, x_-]$ is compatible with both values of θ and leaves uncertainty on which θ has realised.

The conditional p.d.f. of x_m conditional on θ is given by

$$f_{x_m|\theta}(y\,|\,\theta, \mathbf{b}) = \begin{cases} \max\{\overline{b}_{cb} - a\,\underline{b}, 0\} & \text{for } y = x_{max} \\ \frac{a(1+\underline{b})}{\sigma_x}\phi\left(\frac{\theta - y}{\sigma_x}\right) & \text{for } y \in (x_{min}, x_{max}). \end{cases} \tag{31}$$

Note that we are treating the market signal as a random variable on the *extended* real line, that is, including the infinity elements as actual numbers. This is useful to deal with the particular AP policy we are assuming: when $\overline{b}_{cb} > a\,\underline{b}$, the conditional density has a mass point at $x_m = x_{max} = +\infty$, because there is a non-empty set of states in which net supply is zero and the market is passive.[15] When instead $\overline{b}_{cb} < a\,\underline{b}$, the conditional density of x_m becomes a standard truncated normal density with no mass points.

An illustration. Figure 1 illustrates the mapping from fundamentals (θ, S) to the equilibrium marginal investor and price signal entailed by (28). The figure plots the realisation of x_m on the y-axis as a function of the net supply shock S on the x-axis, and the fundamental shock θ: the solid and dashed lines refer to the case where θ is equal to θ_H and θ_L respectively. Each panel illustrates different combinations of $(\underline{b}, a, b_{cb})$.

The left panel, illustrates our benchmark case, $\{\underline{b}, a, b_{cb}\} = \{0, 1, 0\}$. In this case, $\widetilde{S} = S$ and $\mathcal{M} = \mathcal{N}_x$. When $S = 1$, the whole population of investors is needed to clear the market, so the most pessimistic investor, i.e. the one with the lowest private signal ($x_i \to -\infty$), is marginal. When instead $S = 0$, there is an infinitesimal amount of supply, such that only the most optimistic investor ($x_i \to \infty$) buys bonds and is marginal. For a given value of $S \in (0, 1)$, the mass of investors required to clear the market and the marginal investor's identity (or position in the distribution) do not change with θ, but her signal will be more optimistic when θ is high,

[15] It is easy to verify that

$$\int_{\theta_L - \sigma_x \Phi^{-1}\left(\frac{1 + a\,\underline{b} - \overline{b}_{cb}}{a + \underline{b}}\right)}^{+\infty} \frac{a + \underline{b}}{\sigma_x}\phi\left(\frac{\theta - y}{\sigma_x}\right) \mathrm{d}y + (\overline{b}_{cb} - a\,\underline{b}) = 1$$

and thus the integral of the p.d.f. on the extended real line is equal to 1. When instead $\overline{b}_{cb} < a\,\underline{b}$

$$\int_{\theta_L - \sigma_x \Phi^{-1}\left(\frac{1 + a\,\underline{b} - \overline{b}_{cb}}{a(1+\underline{b})}\right)}^{\theta_H - \sigma_x \Phi^{-1}\left(\frac{a\,\underline{b} - \overline{b}_{cb}}{a(1+\underline{b})}\right)} \frac{a(1 + \underline{b})}{\sigma_x}\phi\left(\frac{\theta - y}{\sigma_x}\right) \mathrm{d}y = 1.$$

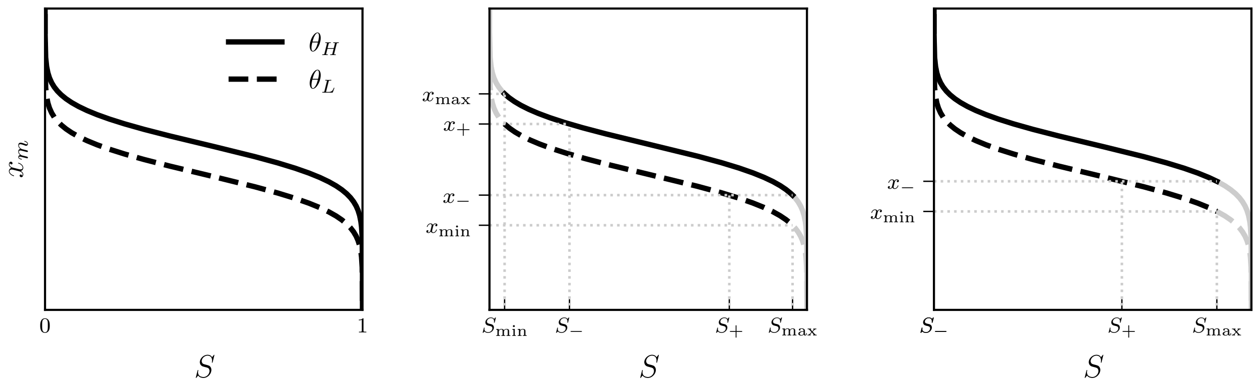

Figure 1: In the right panel, the top (resp. bottom) line plots the realisation of the price signal in case θ^H (resp. θ_L) as a function of *net supply per individual exposure* S. In the left panel, we plot on the x-axis the probability density function of the price signal realisation, which is measured on the y-axis. The left panel plots the benchmark case of unit position bounds and no APs $(\underline{b}, a, \overline{b}_{cb}) = (0, 1, 0)$. The centre panel plots the generic case with short-selling and no APs $(\underline{b}, a, \overline{b}_{cb}) = (-0.05, 1.05, 0)$. The left panel plots a case with APs $(\underline{b}, a, \overline{b}_{cb}) = (-0.05, 1.05, 0.07)$.

because that shifts the mean of the private signal distribution. In other words, $\theta = \theta_H$ as $x_m(\theta_H, S) > x_m(\theta_L, S)$ always.

The central panel illustrates a case with position bounds outside unity, that is, short-selling is possible and long positions can be larger than one: $-\underline{b} < 0, a > 1, b_{cb} = 0$. When short and long position bounds are outside unity, the marginal investor distribution has truncated tails as $S_{min} > 0$ and $S_{max} < 1$. Some very optimistic investors are never marginal, as there is always enough supply from short sellers to satisfy their demand; on the other extreme, some very pessimistic buyers are never marginal either, as long position bounds are such that more optimistic investors are always enough to meet supply.

Finally, the right panel shows the effect of central bank uncontingent asset purchases, keeping the same position bounds of the central panel: $-\underline{b} < 0, a > 0, \overline{b}_{cb} > a\,\underline{b} > 0$. We assume $\overline{b}_{cb} > a\,\underline{b}$ to highlight a difference from the central panel of the figure: sufficiently large APs have the effect of absorbing all the short selling, which implies that we are back in the case where $S_{min} = 0$, zero net supply is a possibility, and the most optimistic investor in the whole population can be marginal. Similarly to the central panel, in the left tail of the marginal agent distribution, there is an interval of investors which are never marginal, as the intervention of the central bank always crowds their purchases out. The presence of a left tail truncation in the marginal agent distribution gives rise to an interval $[x_{min}, x_-)$ of price signals whose observation is uniquely associated with the realization of θ_L.

It is worth noting that any combination of $(\underline{b}, a, \overline{b}_{cb})$ that delivers the same (S_{min}, S_{max}) pairs generates identical truncations of the marginal agent distribution. In this sense,[16] implementing uncontingent asset purchases is analogous to an expansion of the long position limit, since it effectively increases the amount of bonds purchased at any market price, as if investors could absorb a larger stock of assets.

3.3 Marginal Investor Beliefs

In the analysis that follows, we often condition on the market being active because we are interested in characterising the equilibrium price when it is determined by the market, rather than the central bank. This implies that we condition on $x_m \in (x_{min}, x_{max})$, and not on the case where $x_m = x_{max}$ whenever that is a possibility.

3.3.1 "Cursed" Posterior Beliefs: Only Private Signals

It is instructive to first consider the simplest case where agents are "cursed" in the terminology of Eyster et al. (2019), i.e. they condition their beliefs on the private signal (and APs rule) but neglect the information content of the price they observe. In this case, the posterior probability of $\theta = \theta_H$ conditional only on private and prior information is given by

$$p_i^{cur} := P(\theta = \theta_H \,|\, x_i \sim \mathcal{N}_x) = \frac{q\,\phi\left(\frac{\theta_H - x_i}{\sigma_x}\right)}{q\,\phi\left(\frac{\theta_H - x_i}{\sigma_x}\right) + (1-q)\,\phi\left(\frac{\theta_L - x_i}{\sigma_x}\right)},$$

i.e. it is the probability that a certain net supply realisation consistent with the observation of x_i occurred conditional to θ_H rather than θ_L. Therefore $R(p_m^{cur})$, with m defined as the identity of the investor observing the threshold signal $x_i = x_m$ with x_m given by (28), would be the equilibrium return prevailing in a market with cursed investors. We will use the "cursed" case as our simplest benchmark of beliefs formation and market price to isolate the effect of learning from prices.

[16] But not in general, as different combinations of $(\underline{b}, a, \overline{b}_{cb})$ imply different mappings between gross and net supply.

3.3.2 "Public" Posterior Beliefs: Only Price Signals

As an intermediate step, it is instructive to derive the posterior belief conditional on public information only, i.e. the probability of θ_H conditional on the observation of the realisation of the marginal agent (28), equivalent to the observation of the market price. As we highlighted previously, depending on $\{\underline{b}, a, \overline{b}_{cb}\}$ there may be partitions of the price signal support in which the value of the fundamental is fully revealed conditional on observing $x_m \in [x_{min}, x_-) \cup (x_+, x_{max}]$. In this case, the posterior probability of $\theta = \theta_H$ conditional only on public and prior information is given by

$$p_m^{pub} := P(\theta_H \,|\, x_m \sim \mathcal{M}(\mathbf{b})) = \begin{cases} 1 & \text{if} \quad x_m \in (x_+, x_{max}], \\ \dfrac{q\,\phi\left(\frac{\theta_H - x_m}{\sigma_x}\right)}{q\phi\left(\frac{\theta_H - x_m}{\sigma_x}\right) + (1-q)\phi\left(\frac{\theta_L - x_m}{\sigma_x}\right)} & \text{otherwise} \\ 0 & \text{if} \quad x_m \in [x_{min}, x_-), \end{cases}$$

where note $p_m^{pub} = p_m^{cur}$ within the non-revealing region. In fact, in this region, the information given by the price coincides with the information contained in the marginal agent's private signal. In fully revealing regions, however, conditioning on public information will take advantage of the fact that a particular realisation of the price signal x_m is consistent with only a particular realisation of θ. $R(p_m^{ext})$ is thus a "publicly evaluated" interest rate, i.e. it represents the price that would make an external observer using only public information indifferent between buying bonds and selling them short.

3.3.3 Equilibrium Posterior Beliefs: Private Signals and Learning from Prices

We can now characterise the equilibrium posterior beliefs of investor i conditional on her whole information set, which includes: *i)* the AP rule and exogenous prior distributions, *ii)* the private signal x_i, drawn from its distribution $\mathcal{N}_x$; and *iii)* the endogenous public signal given by the market price R, which is equivalent to that contained in x_m. In this case, the posterior probability of $\theta = \theta_H$ conditional on $\Omega_i = \{x_i \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})\}$ is given by

$$
\begin{aligned}
p_{i,m} :=& P(\theta_H \,|\, x_i \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})) = \\
=& \begin{cases}
1 & \text{if} \quad x_m \in (x_+, x_{max}], \\
\dfrac{q\,\phi\left(\frac{\theta_H - \frac{x_i + x_m}{2}}{\sigma_x/\sqrt{2}}\right)}{q\,\phi\left(\frac{\theta_H - \frac{x_i + x_m}{2}}{\sigma_x/\sqrt{2}}\right) + (1-q)\,\phi\left(\frac{\theta_L - \frac{x_i + x_m}{2}}{\sigma_x/\sqrt{2}}\right)} & \text{otherwise} \\
0 & \text{if} \quad x_m \in [x_{min}, x_-),
\end{cases}
\end{aligned}
\tag{32}
$$

where note $p_{i,m} = p_m^{pub}$ within the fully revealing regions, whereas it is different otherwise. It is worth noting that, whenever the price signal is not fully revealing, the precision of the posterior beliefs of an investor observing private and public information is exactly double that of both a cursed investor not updating public information, and an external observer not holding private information. This follows from the fact that, in the non-revealing region, both private and price signal have the same precision, and observing both doubles the precision of posterior beliefs. Importantly, $R(p_{m,m})$ defines the equilibrium return in our model, which obtains by evaluating $p_{i,m}$ with $x_i = x_m$:

$$
R\,\mathbb{E}\left[\frac{\theta}{\Pi} \,|\, x_m \sim \mathcal{N}_x,\, x_m \sim \mathcal{M}(\mathbf{b})\right] = R\,\left[p_{m,m}(x_m)\theta_H \frac{1-\hat{\alpha}}{1-\hat{\alpha}R\theta_H} + (1-p_{m,m}(x_m))\theta_L \frac{1-\hat{\alpha}}{1-\hat{\alpha}R\theta_L}\right] = 1. \tag{33}
$$

3.4 A Characterisation of Equilibrium Returns

To characterise the equilibrium price, it is useful to establish the following statement.

Lemma 1. *When prices are not fully revealing, the equilibrium posterior of the marginal agent is larger than the public posterior of the marginal agent, i.e.* $p_{m,m} > p_m^{cur} = p_m^{pub}$*, if and only if*

$$
x_m > x^* := \frac{\theta_H + \theta_L}{2},
$$

with $x_m \in (x_-, x_+)$*. In particular,* $p_{*,*} = p_*^{cur} = p_*^{pub} = q$ *where* $p_{*,*}$*,* p_*^{pub} *and* p_*^{cur} *represent the equilibrium, public and cursed posterior of the marginal agent, respectively, computed conditional to* x^**.*

Proof. Postponed to appendix 3 ∎

The lemma states that the equilibrium posterior of the marginal agent is above (below) the public one when the price signal is not fully revealing and higher (lower) than the uninformative value x^*.

To gain intuition, consider first the effect of exogenous public news (e.g. investors observing an exogenous public signal on θ) on the equilibrium. A public signal above the prior mean of θ makes investors' beliefs shift up, and the equilibrium interest rate shift down, with equal elasticity, without affecting the relative mass of buyers.[17] In this case, investors do not learn anything from the price change in itself because it is entirely due to the variation in public news. Consider now the effect of a change in the equilibrium price (hence x_m) due to a shock to the fundamental θ. This will shift up the distribution of investors' private signals as well as the equilibrium price, without affecting the mass of buyers. The crucial difference with the previous example is that x_m is an *endogenous* signal that aggregates private information: when investors see the equilibrium price go up, they revise their beliefs up again. This update triggers a further shift up in the market price, in a loop of amplification.

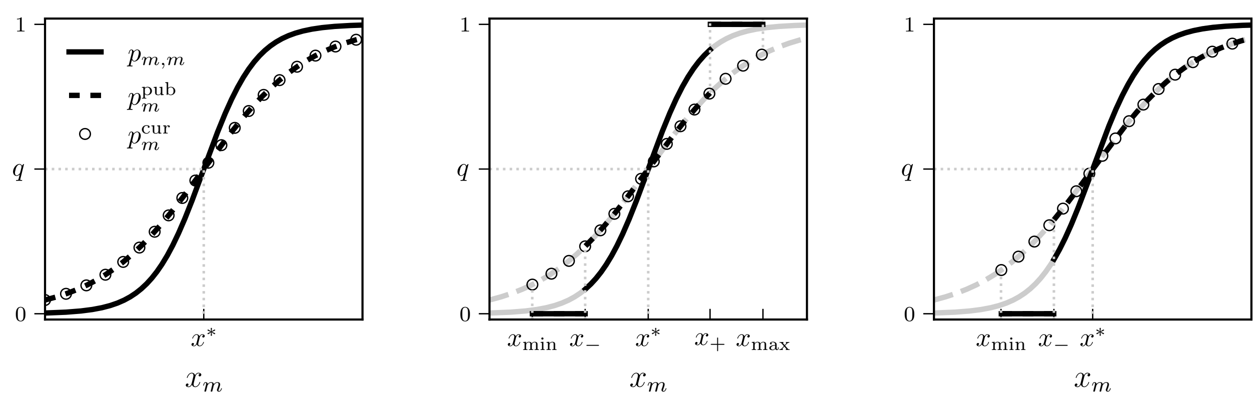

Figure 2: The figure illustrate the relation among cursed, public and equilibrium marginal posteriors as a function of the market signal x_m. The left panel plots the benchmark case of unit position bounds and no APs $(\underline{b}, a, \overline{b}_{cb}) = (0, 1, 0)$; the central panel plots the generic case with short-selling and no APs $(\underline{b}, a, \overline{b}_{cb}) = (-0.05, 1.05, 0)$; the right panel plots a case with APs $(\underline{b}, a, \overline{b}_{cb}) = (-0.05, 1.05, 0.07)$.

Figure 2 illustrates Lemma 1, plotting cursed (with circles), public (with a dashed line) and equilibrium (with a solid line) posterior beliefs of the marginal agent as a function of the realisation of the price (or marginal agent) signal, in the same three specifications of Figure 1. In the first panel, unit position bounds and no APs imply that the private signal and price signal distributions $\mathcal{N}_x$ and $\mathcal{M}$ coincide. The market signal does not generate fully-revealing regions, and the cursed and public posteriors both lie on the same curve $p_m^{curs} = p_m^{pub}$. Cursed, public

[17]This is evident from the market clearing condition that depends solely on the dispersion of investors' beliefs.

and equilibrium posteriors all take value q at x^*, and are strictly monotonically increasing in x_m, ranging from 0 at $x_m \to -\infty$ to 1 at $x_m \to \infty$. At x^*, cursed and public marginal posteriors are flatter than equilibrium ones, meaning that the latter react to market news more than the former.

In analogy to Figure 1, the central panel illustrates a case with position bounds outside $[0, 1]$ and without APs. The presence of truncations in the support of the market signals implies $-\infty < x_{min} < x_{max} < +\infty$, which narrows the range of possible marginal agents. Cursed and public posteriors now differ in the regions of full revelation. Full revelation occurs only for the external observer who takes advantage of the observation of the realisation x_m vis a vis its distribution $\mathcal{M}$, whereas cursed agents do not. In full revelation regions, equilibrium marginal posteriors are equal to public ones, since x_m becomes an infinitely precise signal on θ and all uncertainty is resolved.

Finally, the right panel illustrates the effect of APs, which is essentially to shift the support of x_m towards the right. On the one hand, larger APs make it possible that relatively more optimistic investors can become marginal, as the central bank absorbs all the short sales, and states with zero net bond supply can realise. On the other hand, larger APs imply that relatively more pessimistic investors can never become marginal, as states with large net supply states cannot realise due to APs. All in all, the effect of APs is to shift right towards the right all the bounds $x_{min}, x_-, x_+, x_{max}$, making full revelation more likely to occur for bad states (θ_L) than good ones (θ_H).

Finally, it is instructive to note that, in the non-revealing region (x_-, x_+), the curves of the centre and right panels perfectly overlap with the curves in the left panel (in light gray): this is a property of truncated normal distributions for which the ratio of probability of two outcomes from the same distribution does not change with the size of the truncation, as far as these probabilities do not take degenerate values. In other words, position bounds and APs only affect the support of x_m and its partitions and distribution, and not the posterior distribution of θ conditional on x_m when uncertainty remains.

Average ex-post returns. A consequence of learning from prices is that the equilibrium interest rate does not reflect a fair evaluation of the asset. To appreciate this statement, it is useful to write down the expression for the average ex-post rate of return for bonds

$$\mathbb{E}[R(p_{m,m})\,\theta] = \mathbb{E}[\mathbb{E}[R(p_{m,m})\,\theta \,|\, x_m \sim \mathcal{M}(\mathbf{b})]] = \mathbb{E}\left[\frac{\mathbb{E}[\theta \,|\, x_m \sim \mathcal{M}(\mathbf{b})]}{\mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}_x\,,\, x_m \sim \mathcal{M}(\mathbf{b})]}\right] = \mathbb{E}\left[\frac{R(p_{m,m}(x_m))}{R(p_m^{pub}(x_m))}\right],$$

where, by exploiting the law of iterated expectation, we show that the average ex-post return on bonds is equal to the average ratio between the equilibrium interest rate and its analogous evaluated according to public posterior beliefs. It is immediate to see that, in states where $R(p_m, m) = R(p_m^{pub})$, the ratio is exactly equal to the safe rate of one. These states correspond to fully revealing regions, while the equivalence is generally not true in the whole range of x_m. We can then state the following Lemma.

Lemma 2. *In equilibrium, sufficiently large APs decrease the average ex-post return* $\mathbb{E}[\,R(p_{m,m})\,\theta]$ *strictly below the safe rate of one. In particular,*

$$\frac{\partial \mathbb{E}[\,R(p_{m,m})\,\theta]}{\partial b_{cb}}\Big|_{b_{cb}=a\,\underline{b}} < 0, \qquad \textit{and} \qquad \frac{\partial \mathbb{E}[\,R(p_{m,m})\,\theta]}{\partial b_{cb}}\Big|_{b_{cb}=a(1+\underline{b})} > 0,$$

that is, the amount of uncontingent APs that minimizes $\mathbb{E}[\,R(p_{m,m})\,\theta]$ *is interior in* $(a\,\underline{b}, a(1+\underline{b}))$.

Proof. Postponed in Appendix 4. ■

The Lemma provides a characterization of the impact of asset purchases on the average ex-post market return. It highlights how asset purchases prevent the realization of states where relatively more pessimistic investors become marginal, therefore taking out states in which the market rate is lower than the public one.

4 Monetary-Fiscal Interactions and Welfare

4.1 Individual Rates of Return

Our final goal is to write down savers' and investors' unconditional expectation of the total return on their savings, $\mathbb{E}[\mathcal{R}_s]$ and $\mathbb{E}[\mathcal{R}_i]$ respectively. We are interested in the unconditional version of these expectations for two related reasons. First, because they determine the first-period demand for consumption and savings by savers and investors via Euler equation (17). Second, because these expectations depend on agents' savings decisions and on central bank APs. In fact, an equilibrium of the macroeconomic model is a fixed point of this two-way relationship between prices and allocations.

Savers. In the case of savers, the expected return on money depends on the unconditional distribution of inflation, which in turn depends on the interaction between the treasury and the central bank, and the profits and losses of the latter. We postpone this discussion to Subsections 4.2 and 4.3, where we consider the cases of monetary and fiscal dominance respectively.

Investors. With respect to investors, we can make some progress in characterising the way in which APs affect their expected return from savings, without making specific assumptions about fiscal-monetary interactions. Joining equations (12) and (18), the investors' expected return *per unit* of savings, conditional on her whole information set, is

$$\mathbb{E}\left[\mathcal{R}_i(b_i)\,|\,\Omega_i\right] = b_i(x_m)\mathbb{E}\left[\frac{R\,\theta}{\Pi}\,|\,\Omega_i\right] + (1-b_i(x_m))\,1 \tag{34}$$

where $\Omega_i = \{x_i \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})\}$, and $b_i(x_m)$ is defined in (27).

To derive investors' unconditional expected return from savings, let us first integrate (38) over the private signal distribution, while continuing to condition on public information $x_m \sim \mathcal{M}(\mathbf{b})$:

$$\begin{aligned}
&\mathbb{E}\left[\mathcal{R}_i(b_i)\,|\,x_m \sim \mathcal{M}(\mathbf{b})\right] = \\
&= \int_{-\infty}^{+\infty} \left(b_i(x_m)\,\mathbb{E}\left[\frac{R\,\theta}{\Pi}\,|\,\Omega_i\right] + (1-b_i(x_m))1\right) \mathrm{d}\,F_{x_i|x_m}(x_i|x_m,\mathbf{b}) = \\
&= 1 + \int_{-\infty}^{+\infty} b_i(x_m)\left(\frac{\mathbb{E}[\theta/\Pi\,|\,x_i \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})]}{\mathbb{E}[\theta/\Pi\,|\,x_m \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})]} - 1\right) \mathrm{d}\,F_{x_i|x_m}(x_i|x_m,\mathbf{b})
\end{aligned} \tag{35}$$

where we used the equilibrium price equation (33) to substitute out R, and $F_{x_i|x_m}(x_i)$ is the c.d.f. of the private signal conditional on the market signal, and its p.d.f. is given by

$$f(x_i|x_m,\mathbf{b}) = \frac{1}{\sigma_x}\phi\left(\frac{x_i - x_m}{\sigma_x\sqrt{2}}\right)\frac{\sum\limits_{\theta\in\Theta(x_m,\mathbf{b})} q(\theta)\phi\left(\frac{\frac{x_i+x_m}{2}-\theta}{\frac{\sigma_x}{\sqrt{2}}}\right)}{\sum\limits_{\theta\in\Theta(x_m,\mathbf{b})} q(\theta)\phi\left(\frac{x_m-\theta}{\sigma_x}\right)} \tag{36}$$

where Θ is the set of values of θ that have positive probability conditional on $(x_m, \mathbf{b})$.[18] The last equality of equation (35) highlights two important facts. First, for any x_m we condition upon, the expected return on savings for an investor is bounded below by 1. This is due to the fact that the term

$$\frac{\mathbb{E}[\theta/\Pi\,|\,x_i \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})]}{\mathbb{E}[\theta/\Pi\,|\,x_m \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})]} - 1$$

[18]The derivation can be found in Proof 5 in the appendix.

inside the integral represents the *expected excess return of bonds over the safe asset*, conditional on investor i's private information, and must be non-negative: if $x_i \geq x_m$, the expected excess return is positive, and the investors puts all her savings in bonds (i.e. she takes a long portfolio share equal to 1); if instead $x_i < x_m$, the expected excess return is negative and the investor takes a short position in bonds equal to $-\underline{b} < 0$, and puts her savings plus the revenues from short sales into the safe asset. Second, whenever the price signal is fully revealing, all uncertainty is resolved, bonds and the safe option are equivalent assets with a deterministic payoff, bond excess returns are zero, and the whole integrand in equation (35) cancels out.

Integrating (35) once more with respect to the price signal distribution, we get the unconditional expectation of the return on savings as

$$\mathbb{E}[\mathcal{R}_i(b_i) \,|\, \mathbf{b}] = \int_{\text{Supp}(x_m)} \mathbb{E}\left[\mathcal{R}_i(b_i) \,|\, x_m \sim \mathcal{M}(\mathbf{b})\right] \, \mathrm{d}F_{x_m}(x_m, \mathbf{b})$$

where $F_{x_m}(x_m, \mathbf{b})$ is the marginal c.d.f. of the market signal, whose p.d.f. is given by

$$f_{x_m}(x_m, \mathbf{b}) = \begin{cases} \max\{\bar{b}_{cb} - a\, \underline{b}, 0\} & \text{for } x_m = x_{max} \\ \frac{a(1+\underline{b})}{\sigma_x} \sum_{\Theta(x_m, \mathbf{b})} q(\theta) \phi\left(\frac{x_m - \theta}{\sigma_x}\right) & \text{for } x_m \in (x_{min}, x_{max}). \end{cases} \tag{37}$$

We can split the support of the market signal x_m in two partitions: the interval $[x_-, x_+]$ where uncertainty is never resolved, and the region $(x_{min}, x_-) \cup (x_+, x_{max})$ where the price signal is fully revealing. As we have discussed in the previous paragraph, in the latter region investors get zero excess returns from bonds. We thus integrate expected excess returns only in the former region, where uncertainty remains:

$$\mathbb{E}[\mathcal{R}_i(b_i) \,|\, \mathbf{b}] - 1 = \int_{x_-}^{x_+} \int_{-\infty}^{+\infty} w(x_i, x_m) \left(\frac{\mathbb{E}[\theta/\Pi \,|\, x_i, x_m]}{\mathbb{E}[\theta/\Pi \,|\, x_m, x_m]} - 1 \right) \mathrm{d}\, F_{x_i|x_m}(x_i|x_m, \mathbf{b}) \, \mathrm{d}F_{x_m}(x_m, \mathbf{b}) \tag{38}$$

A few things are worth noting. First, the unconditional expected excess return over the safe asset is non-negative for the reasons illustrated in the previous paragraph. Second, expected returns depend on APs and position bounds (i.e. $\mathbf{b}$) through *(i)* the interval of non-revealing price signals $[x_-, x_+]$, *(ii)* the size of long and short positions investors can take, $w(x_i, x_m, \mathbf{b})$, *(iii)* the marginal distribution of the price signal, F_{x_m}, and *(iv)* the amount by which inflation depends on bond returns when in the fiscal dominance regime, α. Note that, conditional on x_m being in the non-revealing region, the ratio of conditional expectations and the conditional distribution of x_i given x_m do not depend on $\mathbf{b}$.

The next subsections make specific assumptions about fiscal-monetary interactions and the behaviour of inflation, and describe in detail the effect of APs on asset prices and expected returns, savings, consumption and welfare.

4.2 Monetary Dominance

We now assume that the transfer policy between the government and the central bank is such that the gross inflation rate is constant and equal to 1. That is, we assume a transfer policy such that all profits and losses the central bank makes as a result of engaging in APs are fully rebated to the government. In the wording and notation of Section 2.1, we assume full fiscal backing, $\kappa = 1$, and allow α to vary with $\bar{b}_{cb}$.

As a result, the behaviour of savers becomes uninteresting, because the rate of return on their savings is now equal to the first best level, and their savings and consumption decisions are efficient. We thus focus solely on the effect of uncontingent APs on investors' expected rate of return.

In the case of investors, the inflation term disappears from the ratio of conditional expectations in equation (38). In Figure 3, the bottom row represents the inner integral of that equation, i.e. the expected return from savings conditional on x_m. The top row plots the marginal distribution of the market signal x_m. Each column represents a different configuration of position bounds and asset purchases. Computing the unconditional expected return from savings amounts to integrating the product of these two objects over the whole support of the market signal x_m.

In the first column we look at the baseline case with unit position bounds and no APs. The marginal distribution of x_m is symmetric around the prior mean and has support given by the entire real line, since in this case there are no instances of full revelation and support truncations. Expected excess returns are weakly positive, and have an asymmetric distribution that is skewed towards the left. Understanding its shape is not a straightforward exercise, so we now discuss one by one the different mechanisms that drive it. First, the p.d.f. of the private signal conditional on the market signal (see equation (36) moves in the same direction of, but less than, the market signal itself. This implies that the probability investor i is optimistic and buys bond (i.e. $x_i \geq x_m$) is decreasing in x_m. Second, investor i's excess returns $\mathbb{E}[\theta|x_i, x_m]/\mathbb{E}[\theta|x_m, x_m] - 1$ belong to the range $[0,\ \theta_H/\mathbb{E}[\theta|x_m, x_m] - 1]$ as x_i moves inside the $[x_m, +\infty)$ interval. These two observations combined help us explain the behaviour of expected excess returns condition on x_m. When $x_m \to \infty$, excess returns tend to zero: it's less likely the investor will be more optimistic than

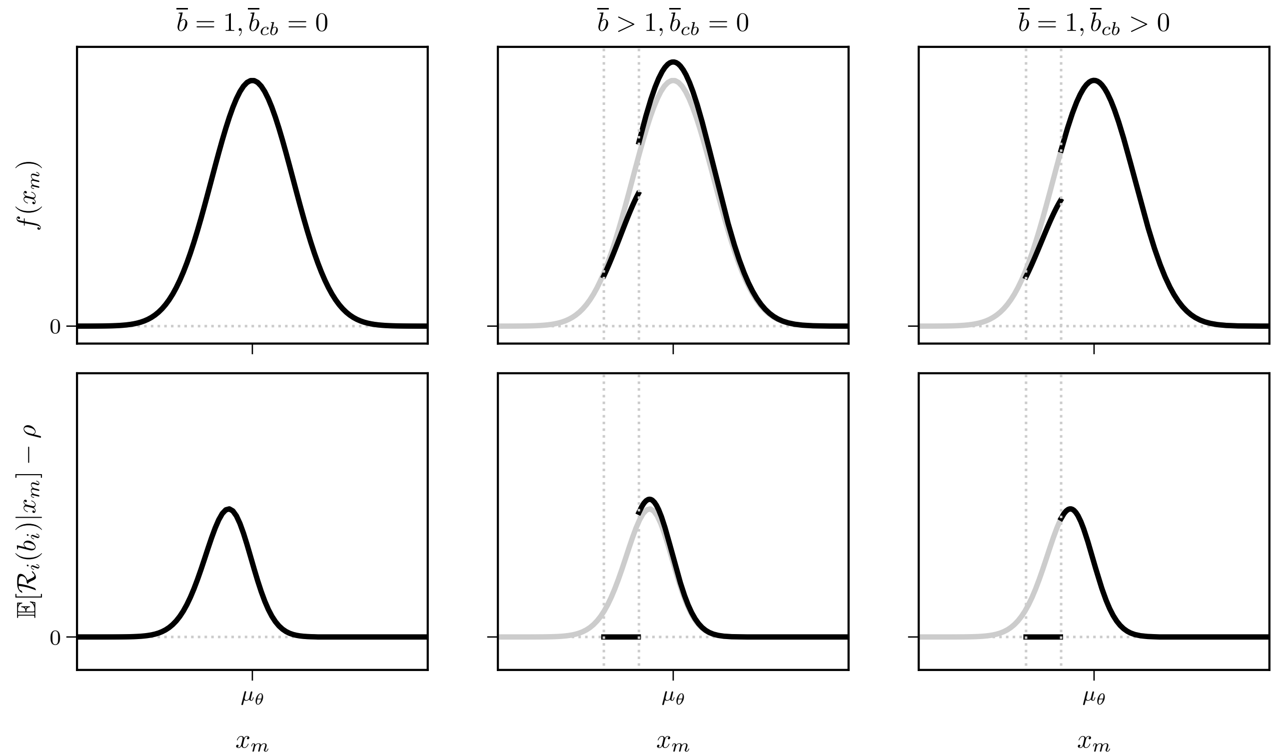

Figure 3: Marginal distribution of the market signal (top row) and expected excess return from saving in bonds (bottom row) as a function of the market signal x_m. The x axis displays values of x_m within 5 standard deviations of its marginal distribution. The light gray lines in the middle and right columns represent the case illustrated in the left column, with unit bounds and no APs.

the marginal agent, and even if she is, there is little upside to be made by buying the bonds if the price is close to its highest value θ_H. As x_m decreases, the probability that investor i receives a signal $x_i \geq x_m$ grows, and the range of excess returns that may realise in such cases becomes wider and populated with higher values: as the equilibrium price drops, there is more upside to be made when receiving a high private signal. But as x_m decreases, so does the density of $x_i|x_m$. When x_m becomes low enough, the probability mass over signals that correspond to high excess returns shrinks towards zero. As $x_m \to -\infty$, expected excess returns converge to zero, as the investor makes positive returns that are significantly above zero only when she receives private signals that are very large and have low probability.

The second column of Figure 3 plots the case where investor savings (i.e. the long position bound) are larger than unity. Larger savings imply the net supply per investor decreases. This shrinks the support of the market signal: all values of x_m to the left of the first vertical dotted line fall out of the support, the values in between the vertical lines are now only compatible with

θ_L, and the remaining values are compatible with both values of θ. This explains why the density of the market signal has a discontinuous jump, and why expected excess bond returns fall to zero in the region where information is fully revealed. As is clear from equation (37), the reduction in the density of x_m inside the fully revealing region is balanced by an increase in probability mass inside the non-revealing region. While the changes in the support and revealing regions of x_m clearly point to a reduction in the expected excess return from savings, the increase in probability mass in the non-revealing partition of x_m goes in the opposite direction.

The third column of Figure 3 plots the case where the central bank is doing non-contingent APs. The effect is very similar to that of an increase in investors' savings, with one important difference: there is no adjustment in the probability density function of the market signal in the non-revealing region. This happens because APs reduce the probability that the market will be active, so rather than observing a "redistribution" of probability mass from the left to the right, this mass simply disappears and goes into the unconditional probability that the market is passive, which is $1 - P_0 = 1 - \bar{b}_{cb}$. This implies that the effect of APs on the expected return from savings is unambiguously negative.

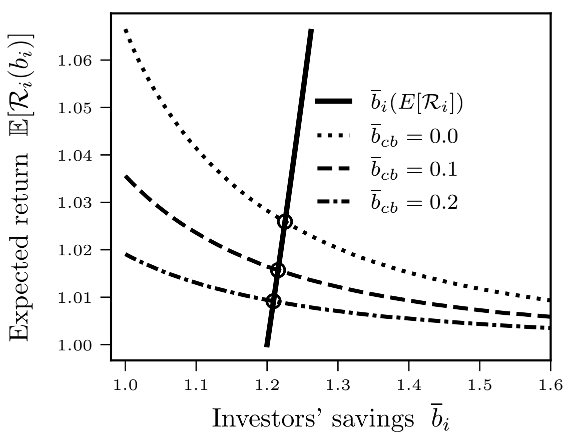

Figure 4: tbc

Equilibrium savings and returns. Having computed $\mathbb{E}[\mathcal{R}_i(b_i) \mid \mathbf{b}]$, we can look for an equilibrium of the economy, which we illustrate in Figure 4. An equilibrium is given by the intersection between two curves and represented by round markers. The first curve, depicted in black, represents investors' supply of savings a_i as a function of their expected return $\mathbb{E}[\mathcal{R}_i \mid \mathbf{b}]$, which can be derived from Euler equation (17), and is increasing and concave. The second set of curves (dotted, dashed and dash-dotted line), represent the expected return from savings $\mathbb{E}[\mathcal{R}_i(b_i) \mid \mathbf{b}]$

as a function of their level a_i and of the central bank asset purchase policy $\bar{b}_{cb}$. We can show that this relationship is decreasing. The smaller the net supply of the bonds, because of higher long position bounds or larger APs, the smaller the excess returns investors expect to make. Importantly, expected returns are decreasing in both position bounds a_i and APs $\bar{b}_{cb}$, which implies that both the equilibrium level and the expected return of savings are decreasing in the central bank asset purchase policy. That is, the higher $\bar{b}_{cb}$, the closer to 1 the returns, and the larger first-period consumption.

In this setting, the optimal AP policy is to set $\bar{b}_{cb}$ as high as possible, to bring returns down to their efficient level of 1, and consumption up to its efficient level of one. This crucially relies on the fact that central bank profits or losses are rebated back to the government, and in turn to the household via lump-sum taxes. We illustrate this aspect in detail in the next paragraph, and then move to consider a setting where central bank losses create a welfare loss, creating a non-trivial trade-off for the central bank.

Central bank profits and losses. Like investors, the central bank makes profits or losses from asset purchases. Before we discuss these in detail, we must specify at what price the central bank is buying when $\bar{b}_{cb} > \widetilde{S} + a\,\underline{b}$ and the market is passive. In such an instance, investors would know the true value of θ because they always observe b_{cb}, and know that it is equal to gross supply whenever it is smaller than its upper bound $\bar{b}_{cb}$. If $\underline{b} > 0$, there will be investors selling the bonds short in the market, and since they observe $\widetilde{S}$, the price must equal θ. We interpret the case where the short-selling bounds are equal to zero as the limit of that where $\underline{b} \to 0$, and so we continue to assume that $R = 1/\theta$ when $\widetilde{S} < \bar{b}_{cb} - a\,\underline{b}$. This implies that, in this scenario, the central bank makes zero profits, because it buys at the full information price.

We can now characterise the expected excess return (over the risk-free rate) of central bank asset purchases. Such return is illustrated in Figure 5 and is formally given by

$$\begin{aligned}\mathbb{E}[b_{cb}\,(R\,\theta - 1)\,|\,\mathbf{b}] &= \bar{b}_{cb}\int_{x_-}^{x_+} \left(\mathbb{E}[R\,\theta\,|\,x_m \sim \mathcal{M}(\mathbf{b})] - 1\right) \mathrm{d}F_{x_m}(x_m, \mathbf{b}) \\ &= \bar{b}_{cb}\int_{x_-}^{x_+} \left(\frac{\mathbb{E}[\theta\,|\,x_m \sim \mathcal{M}(\mathbf{b})]}{\mathbb{E}[\theta\,|\,x_m \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})]} - 1\right) \mathrm{d}F_{x_m}(x_m, \mathbf{b}).\end{aligned} \tag{39}$$

The figure plots the whole integral in equation (39), and depends instead on the equilibrium bond price that arises in the states where the market is active, i.e. when gross bond supply exceeds the $\bar{b}_{cb}$, which happens with probability $1 - P_0$. In these states, the quantity of APs is constant, so profits and losses are solely determined by the behaviour of the expected excess

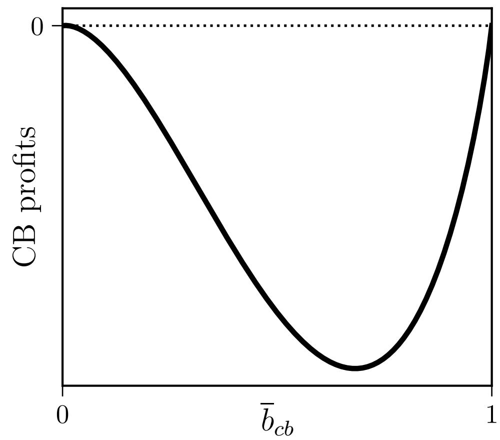

Figure 5: Central bank profits and losses as a function of uncontingent APs $\bar{b}_{cb}$.

return of bonds over the safe asset. As we show in equation (3.4) and the paragraph on ex-post returns, the expression

$$\mathbb{E}[R\,\theta \,|\, x_m \sim \mathcal{M}(\mathbf{b})] - 1 = \frac{\mathbb{E}[\theta \,|\, x_m \sim \mathcal{M}(\mathbf{b})]}{\mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}_x, x_m \sim \mathcal{M}(\mathbf{b})]} - 1 = \frac{R}{R_p} - 1$$

is generally different from zero because of the different conditioning sets. In Figure 2 we show how the posterior probabilities behind these prices differ.[19] From there, we can see that APs create both a truncation and a fully revealing region in the left tail of the support of x_m, where $R/R_p > 1$ and bonds are under-priced. By taking away these states, APs reduce the expected excess return of bonds, so it follows that the central bank makes losses when the market is active.

4.3 Fiscal Dominance

So far, we have developed the analysis under the assumption of monetary dominance, i.e. of a setting of full fiscal backing when central bank profits and losses are fully rebated to the government, and inflation is constant. We now make the opposite assumption and consider a situation without any fiscal backing ($\kappa = 0$). As equation (6) shows, this implies that the rate of return on money (inverse of the gross inflation rate) will be a weighted average of the real return on bonds and on the safe asset, with weight $\frac{b_{cb}}{m}$. Savers' money demand a_s will now depend, through inflation, on central bank APs, the equilibrium bond price, and default. To find the

[19] As we discuss at length in Section 3, the posterior probability of $\theta = \theta_H$ for the marginal agent with some information set is a sufficient statistic for the equilibrium price or interest rate, so we discuss the former rather than the latter.

equilibrium of the two-period model we thus need to solve a two-dimensional fixed point problem: savers' and investors' unconditional expectation of the total return on their savings, $\mathbb{E}[\mathcal{R}_s]$ and $\mathbb{E}[\mathcal{R}_i]$, jointly depend on a_i, a_s and on APs b_{cb}; at the same time, both savings decisions depend on the joint behaviour of the rate of return on bonds and money.

As we saw in the previous subsection, uncontingent APs generate expected losses for the central bank, which are increasing in the size of the purchases. Under fiscal dominance, these losses translate in expected inflation, which is also increasing in the size of the AP program. This reduction in the rate of return of money has a welfare cost, because it decreases savers' incentive to hold money, increasing their first-period consumption above the efficient level. At the same time, APs retain their beneficial effect on the expected rate of return of bonds and on investors' first-period consumption. There is thus a trade-off between increasing investors' welfare and reducing savers', which under some conditions admits an interior solution, implying that the optimal amount of APs is positive and finite.

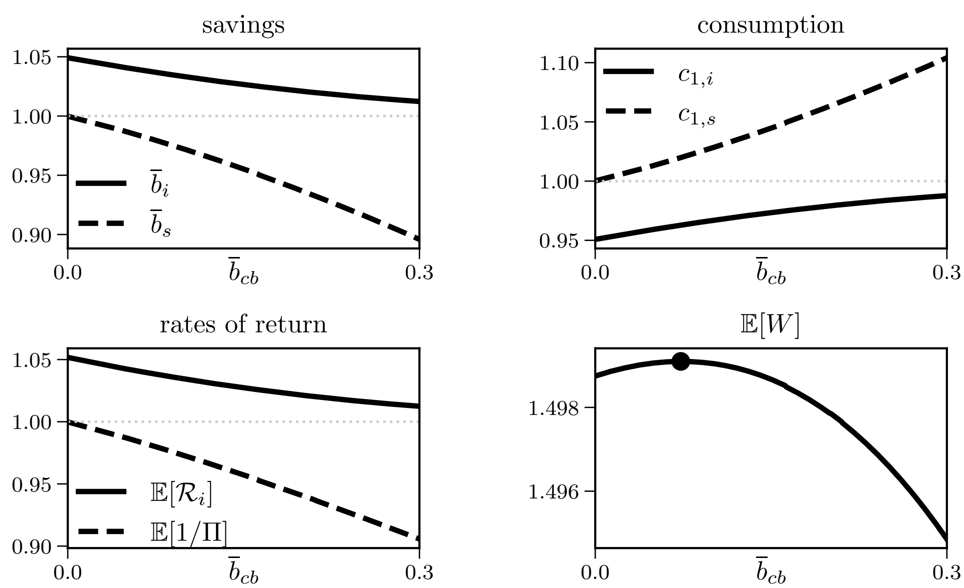

Figure 6: Equilibrium variables under fiscal dominance, as a function of uncontingent APs $\bar{b}_{cb}$. Variables related to investors and savers are represented by solid and dashed lines respectively. Parametric assumptions: $c^* = 1, \gamma = 1$ (log utility)$, q = 1/2, \sigma_x = 1/2, \theta_L = 1/2, \underline{b} = 0$.

Figure 6 illustrates how equilibrium variables depend on the size of APs. The top-left panel plots investors' demand for bonds (which becomes their long position bound in the financial market) and savers' demand for money (which becomes the denominator of $\hat{\alpha}$). The consumption of each set of agents is shown in the top right panel. The bottom-left panel plots the unconditional expectation of the rate of return of bonds and money. As explained, higher APs affect saving demand, rates of return and first-period consumption of investors and savers by bringing each of these variables respectively towards and away from their efficient levels. The optimal uncontingent AP program thus trades off distortions for savers and investors, with the result that aggregate welfare (plotted in the bottom right panel) has an interior maximum at a level $\overline{b}^*_{cb} > 0$.

5 Optimal Price-Targeting Asset Purchases Policy

So far, we have focused the analysis on uncontingent asset purchases, which are a particularly simple type of program where the central bank targets the highest possible price. This allowed us to explain as clearly as possible what are the effects of such policy on consumption, savings, asset prices and welfare.

We have seen that, under fiscal dominance, uncontingent APs cannot eliminate all distortions. We now move on to ask whether an AP policy targeting a different bond price can do better than the optimal uncontingent policy studied in the previous subsection. We have shown that the main channels through which APs have effects are *(i)* the identity of the marginal agent, *(ii)* the information conveyed by the bond price, and *(iii)* the balance sheet of the central bank. We will consider a policy that targets $x_m = x^*$ (and $R_n = R(q)$) for aspect *(i)*, and is neutral in aspects *(ii)*-*(iii)*.

To characterise this policy, we follow three steps: *(i)* understand how an AP policy can target a certain marginal agent x_n; *(ii)* derive the public signal associated with observing a market signal equal to the target x_n; *(iii)* build the mapping between marginal agent x_n and market prices R_n conditional on the given policy.

Implementation and feasibility. Targeting price R_n is equivalent to targeting some marginal agent x_n. Later, we will derive the equilibrium mapping between these two variables. Consider the set of states where the central bank is buying (or is "active") and $R = R_n, x_m = x_n$. Using the market clearing equation (28), we can see that targeting x_n is equivalent to targeting net supply $S_{x_n}(\theta) := \Phi\left(\frac{\theta - x_n}{\sigma_x}\right)$, and we can back out the amount of bonds the central bank is buying

$$b_{x_n}(\theta, \widetilde{S}) = \widetilde{S} + a\,\underline{b} - a(1 + \underline{b})S_{x_n}(\theta). \tag{40}$$

It follows that the equilibrium price is at the target whenever

$$b_{cb} \in [0, \bar{b}_{cb}] \quad \Leftrightarrow \quad \widetilde{S} \in \widetilde{\mathcal{S}}(\theta, x_n) \tag{41}$$

where $\widetilde{\mathcal{S}}(\theta, x_n) := \left[a(1+\underline{b})\, S_{x_n}(\theta, x_n) - a\,\underline{b},\ \bar{b}_{cb} + a(1+\underline{b})\, S_{x_n}(\theta, x_n) - a\,\underline{b}\right]$ denotes the set of gross supply realisations such that APs are feasible and the price is at the target. This set has two important features. First, we assume that x_n, R_n are such that $\widetilde{\mathcal{S}}(\theta, x_n) \subseteq [0,1]$, so the set is always contained in the support of $\widetilde{S}$;[20] second, its length is equal to $\bar{b}_{cb}$ and is independent of θ. Since gross supply is uniformly distributed, the unconditional probability of observing $R = R_n$ and an active central bank is independent of θ and given by

$$\begin{aligned} P(R = R_n,\, b_{cb} \in [0, \bar{b}_{cb}]) &= P(S = S_{x_n}(\theta_H, x_n)) = P(\widetilde{S} \in \widetilde{\mathcal{S}}(\theta_H, x_n)) \\ &= P(S = S_{x_n}(\theta_L, x_n)) = P(\widetilde{S} \in \widetilde{\mathcal{S}}(\theta_L, x_n)). \end{aligned} \tag{42}$$

Information conveyed by the target price. Using the result from (42), it is straightforward to show that the joint observation of a price equal to target and non-zero asset purchases does not convey any information, and the posterior distribution of θ conditional on public information alone is equal to the prior. Formally[21]

$$P(\theta \mid R_n, b_{cb}) = \frac{P(b_{cb} \mid \theta_H, x_n) P(x_n \mid \theta_H) P(\theta_H)}{\sum_{j \in \{H,L\}} P(b_{cb} \mid \theta_j, x_n) P(x_n \mid \theta_j) P(\theta_j)} = q. \tag{43}$$

This implies that the beliefs of investor i conditional on her private information and on price signal $R = R_n$ are given by

$$E[\theta \mid x_i \sim \mathcal{N}_x,\, x_n \sim \mathcal{M}(b_{x_n})] = E[\theta \mid x_i \sim \mathcal{N}_x].$$

Intuitively, this happens because in these states the marginal bond buyer is always the central bank, who is not constrained by its position limits and can absorb any variation in bond supply, making the price is inelastic to it. As we will see below, price become informative when the marginal bond buyer goes back to being an investor again.

[20] This is true if

$$S_{x_n}(\theta, x_n) \in \left[\frac{\underline{b}}{1+\underline{b}},\ \frac{1 + a\,\underline{b} - \bar{b}_{cb}}{a(1+\underline{b})}\right]\ \forall \theta \quad \Leftrightarrow \quad x_n \in \left[\theta_H - \sigma_x \Phi^{-1}\left(\frac{\underline{b}}{1+\underline{b}}\right),\ \theta_L - \sigma_x \Phi^{-1}\left(\frac{1 - \bar{b}_{cb} + a\,\underline{b}}{a(1+\underline{b})}\right)\right].$$

[21] The complete derivation can be found in Proof 7 in the appendix.

Mapping between x_n and R_n. We now have all the elements to define the mapping between the marginal investor x_n and the target bond price R_n, which must satisfy

$$R_n = \frac{1}{\mathbb{E}[\theta \,|\, x_n \sim \mathcal{N}_x,\, x_n \sim \mathcal{M}(b_{x_n})]} = \frac{1}{\mathbb{E}[\theta \,|\, x_n \sim \mathcal{N}_x]}. \tag{44}$$

As usual, the marginal investor is defined as the agent who is indifferent between trading bonds or the safe asset. The last equality uses our finding from the previous step, showing that when the price equals the target and the central bank is active, agents only condition on their private signals. It follows that R_n is defined as the expected bond payoff according to the *public* beliefs of the marginal agent who receives the private signal x_n.

Information and prices away from target. We now consider cases where the price target is not achieved. A direct implication of (41) is that when $\widetilde{S} \notin \mathcal{S}(\theta, x_n)$, then APs are at a corner ($\bar{b}_{cb} \in \{0, \bar{b}_{cb}\}$) and the price is not at the target ($R \neq R_n$). In this set of states, the marginal agent is not x_n: either $R < R_n$ and the central bank limit order is not executed ($b_{cb} = 0$), or $R > R_n$ because APs are constrained by their upper bound ($b_{cb} = \bar{b}_{cb}$). In both cases, the marginal bond buyer is an investor rather than the central bank, the price is elastic to gross supply, agents use the information contained in the price, and the equilibrium price and marginal agent are determined by the following equation, as in the setup with uncontingent APs:

$$R = \frac{1}{\mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}_x,\, x_m \sim \mathcal{M}]} \tag{45}$$

where x_m is given by $x_m(\theta, S(\widetilde{S}, 0))$ as per (28) and (29). That is, the price in this case is defined by the *market* beliefs of the marginal agent with private signal x_m. Note that, exactly as in the case with uncontingent APs, there may be values of x_m which correspond to only one value of the fundamental θ, and thus imply there is full information revelation.

Optimal price target. As illustrated in Figure 2 and the accompanying analysis, the equilibrium prices in (44) and (45) are generated by different belief distributions, and therefore differ from one another in the way described by Lemma 1. When supply is low, $R < R_n$ and the central bank does not intervene. As soon as R reaches and goes above the target, the central bank limit order starts being executed. This implies that the equilibrium price function may jump, because it is not determined by $p_{m,m}$ any more, but by p_m^{pub} instead. If $R_n = R(q)$, then R reaches the target when $x_m = x^*$, at which point $p_m^{pub} = p_{m,m}$. If instead $R_n < (>) R(q)$, then b_{cb}

would have to jump up (down) to a positive (negative) level needed to affect net supply, since posterior probabilities for the public are below (above) those for the market in such a region of the state space. This is one reason why we consider $R(q)$ as a target, the other being its budget neutrality which we analyse next.

Illustration. Figure 7 shows an example of a price-targeting policy where $\bar{b}_{cb}$ is large enough that it never binds. In all panels, dashed and solid line represent θ_L and θ_H respectively, gray lines plot the case without any APs, and gross supply $\widetilde{S}$ is on the x-axis. The left panel plots how much of the central bank limit order is executed; the central panel shows the marginal agent signal; and the right panel illustrates the equilibrium interest rate. The vertical gray dotted lines highlight the intervals of gross supply realisations for which the policy is active and the price is at the target. Outside of this set, either APs are zero and $R < R_n$, or APs are at the upper bound and $R > R_n$. In this particular case, the upper bound on APs is defined by the highest possible purchase the central bank is making when $\theta = \theta_H$ and $\widetilde{S} = 1$. This creates a bound an APs, because if the central bank purchased a larger quantity when $\theta = \theta_L$, that would perfectly reveal the fundamental. As a result, when there is a default and gross supply is above its feasible set, there is full information revelation and R jumps to its highest value.

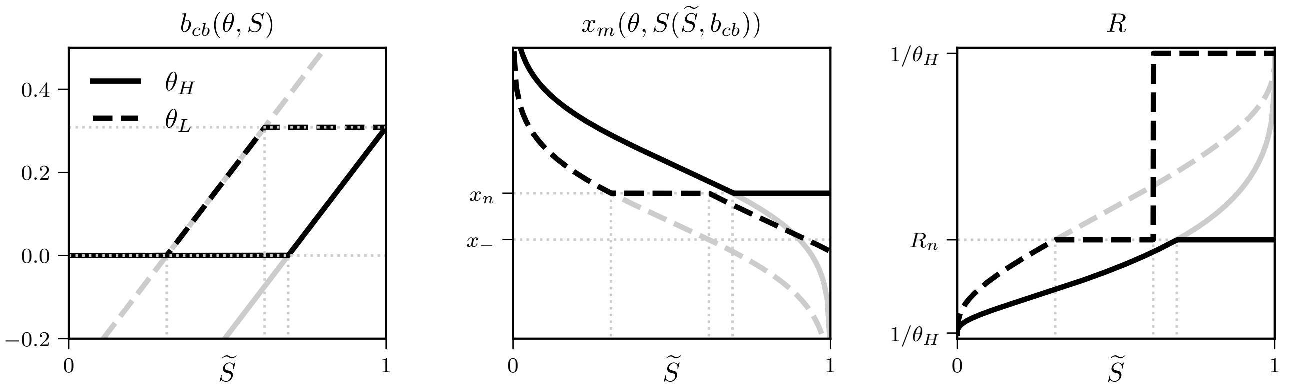

Figure 7:

Central bank profits. We now have all the elements to ask what happens to the central bank balance sheet, inflation, investors' and savers' rates of return, and welfare. When $R_n = R(q)$, the central bank makes profits or losses $\theta - R(q)$ ex post, but in expectation these are always zero: if $\widetilde{S} \in \mathcal{S}(\theta, x_n)$, the central bank buys a varying quantity of bonds at the actuarially fair price, so expected profits are $\mathbb{E}[R\,\theta] - 1 = 0$; if APs are at the upper bound, there is full revelation, and expected profits must be zero because there is no uncertainty; in the remaining states, APs are zero. We have thus proved that

1. price-targeting policies are belief-neutral,

2. a policy with target $R_n = R(q)$ and upper bound $\overline{b}_{cb} \geq b_{x^*}(\theta_H, 1)$ is central-bank-budget-neutral.

The consequence for inflation is that, while some inflation or deflation will happen ex-post, the ex-ante value of expected gross inflation will be one, which means that savers' consumption-savings decision are efficient.

6 Conclusion

TBC

Appendix

A Irrelevance benchmark

We show that, when investors are unconstrained in their asset positions or have homogeneous information, asset purchases are irrelevant as in Wallace (1981). To so, we assume a regime of monetary dominance and consider the portfolio allocation problem of an agent with a generic increasing (and possibly concave) utility function $u(c_{i,2})$, generic bounds $[\underline{b}, \overline{b}]$ on her portfolio choice, and information set Ω_i which contains the AP quantity b_{cb}. Her optimal portfolio choice is given by

$$b_i^* = \arg \max_{b_i \in [\underline{b}, \overline{b}]} \mathbb{E}\left[u\left(a\left[b_i R\, \theta + (1 - b_i)1\right] - T_i\right) \mid \Omega_i\right].$$

Let us take the first-order condition with respect to b_i, and use the government and central bank budget constraints (2) and (4) to replace taxes. We get

$$\mathbb{E}\left[u'\left(\left(R\,\theta - 1\right)\left(a\, b_i + b_{cb} - \widetilde{S}\right) + 1 - e_{cb} - \widetilde{S}\right) \mid \Omega_i\right] + \underline{\mu} - \overline{\mu} = 0 \tag{46}$$

where $\underline{\mu}, \overline{\mu}$ are the Lagrange multipliers of the position bounds constraints $b_i \geq \underline{b}$ and $b_i \leq \overline{b}$ respectively. Equation (46) shows how investors' second period consumption depends on their portfolio choice b_i, government (exogenous) debt policy $\widetilde{S}$, and central bank AP policy b_{cb}. Importantly, the equation highlights how the agent only cares about her *net* exposure to default risk, which is given by her own demand $a\,b_i$, minus the exposure implicit in taxes, which depends on net public sector liabilities $\widetilde{S} - b_{cb}$. Since gross supply is not observed, the investor will set b_i such that her observable exposure $a\,b_i + b_{cb}$ satisfies her first-order condition.

Unconstrained asset positions. Consider first the case where position bounds never bind, and therefore $\underline{\mu} = \overline{\mu} = 0$ for all investors $i \in [0,1]$. This implies that the unconstrained version of equation (46) holds with equality for each investor, who will adjust her individual demand one-for-one with APs, to keep her desired net observable exposure constant.

It follows that, in this setting, APs are neutral because they perfectly crowd out each investor in the same way. Mathematically, $\dfrac{\partial b_i^*}{\partial b_{cb}} = 1$ for all i. This is very different from what happens in the main text, where all investors are constrained by their position bounds, and APs crowd out a specific part of the investor distribution, namely the most pessimistic agents among those that would buy bonds absent APs.

Homogeneous information. Consider now the case where position bounds may bind, and all agents share the same information, $\Omega_i = \Omega$ for all i. This implies that the equilibrium bond price is determined by the optimality condition of a representative investor, and the position bound constraints must not be binding. We thus go back to the case where the unconstrained version of equation (46) holds with equality for each investor, and APs perfectly crowd everyone out by the same amount, resulting in neutrality.

B Proofs

Proof 1 (Proposition 1)**.** *With homogeneous investors, for the bond market to clear at any instances two cases are possible. First, it has to be that* $R\mathbb{E}[\theta/\Pi] = 1$ *so that all investors buy realized supply* $b_i = \widetilde{S}$*. In this case, it is also true that* $\mathbb{E}[\mathcal{R}_i(\widetilde{S})|\bar{\Omega}] = 1$ *and* $\mathbb{E}[\Pi|\bar{\Omega}] = 1$ *because of* (12) *and* (6)*. The second case is when clearing obtains with randomisations so that a commonly observed sunspot coordinates the fraction of buyers needed to clear the market at any state* $(\theta, \widetilde{S})$*. However, such a sunspot must be perfectly correlated with the state* $(\theta, \widetilde{S})$*, so investors will trade under perfect information as a by-product. Under perfect information,* $R\theta/\Pi = 1$ *so that* $\mathcal{R}_i(\widetilde{S}) = 1$ *and* $\Pi = 1$ *at any* $(\theta, \widetilde{S})$ *because of* (12) *and* (6)*.*

Proof 2 (Lemma 2)**.** *Let p denote the probability that $\theta = \theta_H$ for the marginal agent; then the market clearing price is given by the following equation*

$$\left(p \frac{(1-\hat{\alpha})}{\frac{1}{\theta_H} - \hat{\alpha} R} + (1-p) \frac{(1-\hat{\alpha})}{\frac{1}{\theta_L} - \hat{\alpha} R} \right) R = 1,$$

where $\hat{\alpha} := (1-\kappa)\alpha$. The fix point equation can be rewritten as

$$\frac{(R\hat{\alpha}\theta_H - 1)(R\hat{\alpha}\theta_L - 1)}{\theta_L + p\theta_H - p\theta_L - R\hat{\alpha}\theta_H\theta_L} - R(1-\hat{\alpha}) = 0$$

or, provided $\theta^e := \theta_L + p\theta_H - p\theta_L \neq 0$ and $\theta^p := \theta_H\theta_L \neq 0$,

$$(-\hat{\alpha}\theta^p) R^2 + ((1-\hat{\alpha})\theta^e + \hat{\alpha}\theta^s) R - 1 = 0,$$

with $\theta^s := \theta_L + \theta_H$. The only solution positive and smaller than $1/\theta_L$ is

$$R(p) = \frac{(1-\hat{\alpha})\theta^e + \theta^s\hat{\alpha} - \sqrt{((1-\hat{\alpha})\theta^e + \hat{\alpha}\theta^s)^2 - 4\hat{\alpha}\theta^p}}{2\hat{\alpha}\theta^p}$$

where we can show $R(1) = 1/\theta_H$, $R(0) = 1/\theta_L$. In fact, one can verify that

$$\begin{aligned}
\frac{(1-\alpha)\theta^e + \theta^s\alpha - \sqrt{((1-\alpha)\theta^e + \alpha\theta^s)^2 - 4\alpha\theta^p}}{2\alpha\theta^p} &< \frac{1}{\alpha} \\
(1-\alpha)\theta^e + \theta^s\alpha - 2\theta^p &< \sqrt{((1-\alpha)\theta^e + \alpha\theta^s)^2 - 4\alpha\theta^p} \\
((1-\alpha)\theta^e + \theta^s\alpha - 2\theta^p)^2 - ((1-\alpha)\theta^e + \alpha\theta^s)^2 + 4\alpha\theta^p &< 0 \\
4\theta^p(\alpha + \theta^p - \theta^s\alpha + \theta^e\alpha - \theta^e) &< 0 \\
\alpha - \theta^e + \theta^p + \theta^e\alpha - \theta^s\alpha &< 0 \\
\alpha - (\theta_L + p(\theta_H - \theta_L)) + \theta_H\theta_L + (\theta_L + p(\theta_H - \theta_L))\alpha - (\theta_H + \theta_L)\alpha &< 0 \\
\alpha + p\theta_L - \theta_L + p\alpha\theta_H - p\theta_H - \alpha\theta_H - p\alpha\theta_L + \theta_H\theta_L &< 0 \\
\alpha + p\theta_L - \theta_L + p\alpha - p - \alpha - p\alpha\theta_L + \theta_L &< 0 \\
-p(1-\alpha)(1-\theta_L) &< 0.
\end{aligned}$$

The derivative of the equilibrium return with respect to p is given by

$$\frac{\partial R}{\partial p}=-\frac{\left(\theta_H-\theta_L\right)\left(1-\hat{\alpha}\right)}{\sqrt{\left((1-\hat{\alpha})\theta^e+\hat{\alpha}\theta^s\right)^2-4\hat{\alpha}\theta^p}}R<0.$$

By using L'Hopital rule at $\hat{\alpha}=0$ *one can show* $\lim_{\hat{\alpha}\to 0}R=\frac{1}{\theta^e}$*. Moreover, we have*

$$\frac{\partial R}{\partial \hat{\alpha}}=\frac{1}{\hat{\alpha}}\frac{\theta^e R-1}{\sqrt{\left((1-\hat{\alpha})\theta^e+\hat{\alpha}\theta^s\right)^2-4\hat{\alpha}\theta^p}}<0,$$

since

$$\frac{(1-\alpha)\theta^e+\theta^s\alpha-\sqrt{\left((1-\alpha)\theta^e+\alpha\theta^s\right)^2-4\alpha\theta^p}}{2\alpha\theta^p}<\frac{1}{\theta^e},$$

holds if and only if

$$\begin{aligned}
(1-\alpha)\theta^{2e}+\theta^e\theta^s\alpha-\theta^e\sqrt{\left((1-\alpha)\theta^e+\alpha\theta^s\right)^2-4\alpha\theta^p} &< 2\alpha\theta^p \\
(1-\alpha)\,\theta^{2e}+\theta^e\theta^s\alpha-2\alpha\theta^p &< \theta^e\sqrt{\left((1-\alpha)\theta^e+\alpha\theta^s\right)^2-4\alpha\theta^p} \\
\left((1-\alpha)\,\theta^{2e}+\theta^e\theta^s\alpha-2\alpha\theta^p\right)^2 &< \theta^{2e}\left(\left((1-\alpha)\theta^e+\alpha\theta^s\right)^2-4\alpha\theta^p\right) \\
\left((1-\alpha)\,\theta^{2e}+\theta^e\theta^s\alpha-2\alpha\theta^p\right)^2-\theta^{2e}\left((1-\alpha)\theta^e+\alpha\theta^s\right)^2+4\alpha\theta^p\theta^{2e} &< 0 \\
4\theta^p\alpha^2\left(\theta^p+\theta^{2e}-\theta^{s+e}\right) &< 0 \\
\theta^p+\theta^{2e}-\theta^{s+e} &< 0, \\
\theta_H\theta_L+\left(\theta_L+p\left(\theta_H-\theta_L\right)\right)^2-\left(\theta_L+p\left(\theta_H-\theta_L\right)\right)\left(\theta_H+\theta_L\right) &< 0,
\end{aligned}$$

or

$$-p\left(\theta_H-\theta_L\right)^2\left(1-p\right)<0,$$

where notice $Var_p\left(\theta\right)=p\left(\theta_H-\theta_L\right)^2\left(1-p\right)>0$ *always.*

Proof 3 (Lemma 1)**.** *The repayment probability held by agent i holding an information set* Ω_i *is*

$$P(\theta^H|x_i,\,x_m)=\frac{q\,\phi\left(\frac{\theta^H-\left(\frac{\sigma_m^2}{\sigma_x^2}x_i+\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)}{\sigma_m}\right)}{q\,\phi\left(\frac{\theta^H-\left(\frac{\sigma_m^2}{\sigma_x^2}x_i+\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)}{\sigma_m}\right)+(1-q)\,\phi\left(\frac{\theta^L-\left(\frac{\sigma_m^2}{\sigma_x^2}x_i+\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)}{\sigma_m}\right)} \tag{47}$$

whereas the one held by a public observer is

$$P(\theta^H|\, x_m) = \frac{q\,\phi\left(\frac{\theta^H-\left(\frac{\sigma_p^2}{\sigma_x^2}x_m+\frac{\sigma_p^2}{\sigma_y^2}y\right)}{\sigma_p}\right)}{q\,\phi\left(\frac{\theta^H-\left(\frac{\sigma_p^2}{\sigma_x^2}x_m+\frac{\sigma_p^2}{\sigma_y^2}y\right)}{\sigma_p}\right)+(1-q)\,\phi\left(\frac{\theta^L-\left(\frac{\sigma_p^2}{\sigma_x^2}x_m+\frac{\sigma_p^2}{\sigma_y^2}y\right)}{\sigma_p}\right)} \tag{48}$$

where

$$\sigma_m^2 := \frac{1}{2\,\sigma_x^{-2}+\sigma_y^{-2}} \quad \textit{and} \quad \sigma_p^2 := \frac{1}{\sigma_x^{-2}+\sigma_y^{-2}}$$

denote the conditional standard deviation of investors and public observers respectively.

We note that

$$P(\theta^H|x_i = x_m,\, x_m) = \frac{q}{q+(1-q)\,\kappa_m}$$

where

$$\kappa_m := \frac{\phi\left(\frac{\theta^L-\left(2\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)}{\sigma_m}\right)}{\phi\left(\frac{\theta^H-\left(2\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)}{\sigma_m}\right)} = e^{-\frac{1}{2\sigma_m^2}\left(\left(\theta^L-\left(2\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)\right)^2-\left(\theta^H-\left(2\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)\right)^2\right)} =$$

$$= e^{\frac{1}{2\sigma_m^2}(\theta_H-\theta_L)\left(\theta_H+\theta_L-2\left(2\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)\right)}.$$

Performing analogous computations, we can write:

$$P(\theta^H|\, x_m) \;=\; \frac{q}{q+(1-q)\,\kappa_p},$$

where

$$\kappa_p = e^{\frac{1}{2\sigma_p^2}(\theta_H-\theta_L)\left(\theta_H+\theta_L-2\left(\frac{\sigma_p^2}{\sigma_x^2}x_m+\frac{\sigma_p^2}{\sigma_y^2}y\right)\right)}$$

from which the limit statements can be easily proved. Then we verify that x^* is the solution to

$$\frac{1}{2\sigma_m^2}(\theta_H-\theta_L)\left(\theta_H+\theta_L-2\left(2\frac{\sigma_m^2}{\sigma_x^2}x_m+\frac{\sigma_m^2}{\sigma_y^2}y\right)\right) = \frac{1}{2\sigma_p^2}(\theta_H-\theta_L)\left(\theta_H+\theta_L-2\left(\frac{\sigma_p^2}{\sigma_x^2}x_m+\frac{\sigma_p^2}{\sigma_y^2}y\right)\right)$$

which happens when $x^* = (\theta_H + \theta_L)/2$ *at which*

$$\kappa_p = \kappa_m = e^{\frac{1}{\sigma_y^2}(\theta_H - \theta_H)\left(\frac{\theta_H+\theta_L}{2} - y\right)}.$$

As a result,

$$P(\theta^H | x_i = x_m^*, x^*, y) = P(\theta^H | x^*, y) = \frac{q}{q + (1-q) e^{(\theta_H - \theta_L)\left(\frac{\theta_H+\theta_L}{2} - y\right)\frac{1}{\sigma_y^2}}},$$

which are equal to the prior q *in the limit* $\sigma_y^2 \to \infty$*.*

For $x_m < x^*$ *it is easy to check that the right hand side is larger than the left hand side, which proves the inequality statement.*

Finally, the maximal distance between the two posteriors obtains when

$$\frac{\partial P(\theta^H | x_i = x_m,\, x_m)}{\partial x_m} = \frac{\partial P(\theta^H |\, x_m)}{\partial x_m} \quad \Rightarrow \quad \frac{-q(1-q)\frac{\partial \kappa_m}{\partial x_m}}{(q + (1-q)\kappa_m)^2} = \frac{-q(1-q)\frac{\partial \kappa_p}{\partial x_m}}{(q + (1-q)\kappa_p)^2}$$

and since

$$\frac{\partial \kappa_m}{\partial x_m} = -\frac{2}{\sigma_x^2}(\theta_H - \theta_L)\kappa_m \quad \text{and} \quad \frac{\partial \kappa_p}{\partial x_m} = -\frac{1}{\sigma_x^2}(\theta_H - \theta_L)\kappa_p$$

we get the two solutions x^+ *and* x^- *as solution to*

...

Proof 4 (Lemma 2)**.** *We can rewrite the average ex-post equilibrium return as*

$$\mathbb{E}[\, R(p_{m,m})\, \theta\,] \;\; = \;\; \Big[\, P + (1-P) \,\Big(\int \underbrace{\int_{S_*(\theta)}^{S_+(\theta)} \frac{R(p_m(S,\theta))}{R(p_{m,m}(S,\theta))}\, dS}_{>1} + \underbrace{\int_{S_-(\theta)}^{S_*(\theta)} \frac{R(p_m(S,\theta))}{R(p_{m,m}(S,\theta))}\, dS}_{<1} \Big) d\theta \Big]$$

where P *denotes the cumulative probability that* $x_m(\theta, S)$ *is fully revealing of* θ*.*

By construction, both $S_-(\theta)$ *and* $S_+(\theta)$ *strictly decrease (i.e.* $x_-(\theta)$ *and* $x_+(\theta)$*) with* b_{cb} *for any* θ*. The proof obtains since at* $b_{cb} = a\,\underline{b}$*, for given* θ*, the upper boundary of the non revealing region* S_- *(resp.* x_+*) reaches its minimum* $S_- = 0$ *(resp. its maximum* $x_+ \to +\infty$*), so that marginal increases in* b_{cb} *at* $a\,\underline{b}$ *will only shrink the range of* S *where* $R(p_m) > R(p_{m,m})$ *without affecting the one where* $R(p_m) < R(p_{m,m})$*. The second part of the prof obtains since, for given* θ*, at the limit* $S_- \to S_+ \to 0$ *(resp.* $x_- \to x_+ \to +\infty$*), which we get as* $b_{cb} \to a(1 + \underline{b})$

it is $\mathbb{E}[\,R(p_{m,m})\,\theta\,] \to 1$*, and marginal decreases in* b_{cb} *at* $\underline{b}$ *will enlarge the range of* S *where* $R(p_m) < R(p_{m,m})$ *pushing* $\mathbb{E}[\,R(p_{m,m})\,\theta\,]$ *below the natural rate of one.*

Proof 5 (Derivation of the p.d.f. of x_i conditional on x_m in equation (36))**.**

$$
\begin{aligned}
f(x_i|x_m) &= \sum_j q_j f(x_i|x_m,\theta_j) = \sum_j q_j f(x_i|\theta_j) f(x_m|\theta_j) \frac{1}{f(x_m)} = \\
&= \sum_j q_j \frac{a(1+\underline{b})}{\sigma_x} \phi\left(\frac{\theta_j - x_i}{\sigma_x}\right) \frac{1}{\sigma_x} \phi\left(\frac{\theta_j - x_m}{\sigma_x}\right) \frac{1}{f(x_m)} \\
&= \sum_j q_j \frac{a(1+\underline{b})}{\sigma_x/\sqrt{2}} \phi\left(\frac{\theta_j - \frac{x_i+x_m}{2}}{\sigma_x/\sqrt{2}}\right) \frac{1}{\sigma_x\sqrt{2}} \phi\left(\frac{x_i - x_m}{\sigma_x\sqrt{2}}\right) \frac{1}{f(x_m)} \\
&= \frac{1}{\sigma_x\sqrt{2}} \phi\left(\frac{x_i - x_m}{\sigma_x\sqrt{2}}\right) \sum_j q_j \frac{a(1+\underline{b})}{\sigma_x/\sqrt{2}} \phi\left(\frac{\theta_j - \frac{x_i+x_m}{2}}{\sigma_x/\sqrt{2}}\right) \frac{1}{\sum_j q_j \frac{a(1+\underline{b})}{\sigma_x} \phi\left(\frac{x_m-\theta_j}{\sigma_x}\right)} \\
&= \frac{1}{\sigma_x} \phi\left(\frac{x_i - x_m}{\sigma_x\sqrt{2}}\right) \sum_j q_j \phi\left(\frac{\theta_j - \frac{x_i+x_m}{2}}{\sigma_x/\sqrt{2}}\right) \frac{1}{\sum_j q_j \phi\left(\frac{x_m-\theta_j}{\sigma_x}\right)}
\end{aligned}
$$

Proof 6 (Derivations behind equation (38))**.**

$$
\begin{aligned}
\mathbb{E}[\mathcal{R}_i(b_i)] = &\int_{x_m \in [x_{min}, x_-] \cup [x_+, x_{max}]} f(x_m)\, dx_m + \\
&+ \int_{x_-}^{x_+} \int_{-\infty}^{+\infty} [w(x_i, x_m)\, \mathbb{E}[R\,\theta\,|\,\Omega_i] + (1 - w(x_i, x_m))1] \;\, dF(x_i|x_m)\;\, dF(x_m) = \\
&= \left[P_H + P_L + \int_{x_-}^{x_+} \int_{-\infty}^{+\infty} f(x_i|x_m)\, d\,x_i\, dF_{x_m}(x_m)\right] + \\
&+ \int_{x_-}^{x_+} \int_{-\infty}^{x_m} \underline{b}\,(\mathbb{E}[R\,\theta\,|\,\Omega_i] - 1)\; dF_{x_i|x_m}(x_i)\; dF_{x_m}(x_m) + \\
&+ \int_{x_-}^{x_+} \int_{x_m}^{+\infty} (\mathbb{E}[R\,\theta\,|\,\Omega_i] - 1)\; dF_{x_i|x_m}(x_i)\; dF_{x_m}(x_m). \\
&= 1 + \int_{x_-}^{x_+} \int_{-\infty}^{+\infty} (\mathbb{1}[x_i \geq x_m]1 + \mathbb{1}[x_i < x_m]\underline{b}) \Big(\mathbb{E}[R\,\theta\,|\,\Omega_i] - 1\Big) dF_{x_i|x_m}(x_i)\; dF_{x_m}(x_m). \\
&+ \int_{x_-}^{x_+} \int_{-\infty}^{x_m} \underline{b}\,(\mathbb{E}[R\,\theta\,|\,\Omega_i] - 1)\; dF_{x_i|x_m}(x_i)\; dF_{x_m}(x_m) + \\
&+ \int_{x_-}^{x_+} \int_{x_m}^{+\infty} (\mathbb{E}[R\,\theta\,|\,\Omega_i] - 1)\; dF_{x_i|x_m}(x_i)\; dF_{x_m}(x_m).
\end{aligned}
$$

Proof 7 (Derivation of equation (43))**.** *We use Bayes' law to write*

$$\begin{aligned}
P(\theta_H|x_n, b_{cb}) &= \frac{P(b_{cb}|\theta_H, x_n)P(x_n|\theta_H)P(\theta_H)}{\sum_{j\in\{H,L\}} P(b_{cb}|\theta_j, x_n)P(x_n|\theta_j)P(\theta_j)} \\
&= \frac{P\left(\widetilde{S} \in \widetilde{\mathcal{S}}(\theta_H, x_n) : b_{x_n}(\theta_H, \widetilde{S}) = b_{cb}\right) P\left(S = S_{x_n}(\theta_H, x_n)\right) P(\theta_H)}{\sum_{j\in\{H,L\}} P\left(\widetilde{S} \in \widetilde{\mathcal{S}}(\theta_j, x_n) : b_{x_n}(\theta_j, \widetilde{S}) = b_{cb}\right) P(S = S_{x_n}(\theta_j, x_n))\, P(\theta_j)} \\
&= \frac{P(\widetilde{S} = b_{cb} + a(1+\underline{b})S_{x_n}(\theta_H, x_n))\, P\left(S = S_{x_n}(\theta_H, x_n)\right) P(\theta_H)}{\sum_{j\in\{H,L\}} P(\widetilde{S} = b_{cb} + a(1+\underline{b})S_{x_n}(\theta_j, x_n))\, P\left(S = S_{x_n}(\theta_j, x_n)\right) P(\theta_j)} \\
&= \frac{P(\widetilde{S} = b_{cb} + a(1+\underline{b})S_{x_n}(\theta_H, x_n))\, P\left(S = S_{x_n}(\theta_H, x_n)\right) P(\theta_H)}{P(\widetilde{S} = b_{cb} + a(1+\underline{b})S_{x_n}(\theta_H, x_n))\, P\left(S = S_{x_n}(\theta_H, x_n)\right) \sum_{j\in\{H,L\}} P(\theta_j)} = q.
\end{aligned}$$

The last step exploits the result in (42), and implies most terms cancel out. In the previous steps, we simply use the definition of b_{x_n} from (40) and that of $\mathcal{S}(\theta, x_n)$.

References

Albagli, Elias, Christian Hellwig, and Aleh Tsyvinski, "Dispersed Information and Asset Prices," CEPR Discussion Papers 15644, C.E.P.R. Discussion Papers 2021.

Altavilla, Carlo, Domenico Giannone, and Michele Lenza, "The Financial and Macroeconomic Effects of the OMT Announcements," *International Journal of Central Banking*, September 2016, *12* (3), 29–57.

_ **, Giacomo Carboni, and Roberto Mottoa**, "Asset Purchase Programs and Financial Markets: Lessons from the Euro Area," *International Journal of Central Banking*, 2021.

Arrata, William, Benoît Nguyen, Imène Rahmouni-Rousseau, and Miklos Vari, "The scarcity effect of QE on repo rates: Evidence from the euro area," *Journal of Financial Economics*, 2020, *137* (3), 837–856.

Auclert, Adrien, "Monetary Policy and the Redistribution Channel," *American Economic Review*, June 2019, *109* (6), 2333–2367.

Bassetto, Marco and Carlo Galli, "Is Inflation Default? The Role of Information in Debt Crises," *American Economic Review*, 2019, *109* (10), 3556–3584.

Bernanke, Ben. S., "Five Questions about the Federal Reserve and Monetary Policy," url=http://www.federalreserve.gov/newsevents/speech/bernanke20121001a.htm 10 2012. Speech at the Economic Club of Indiana, Indianapolis, Indiana.

Bhattarai, Saroj and Christopher J. Neely, "An Analysis of the Literature on International Unconventional Monetary Policy," Working Papers 2016-021, Federal Reserve Bank of St. Louis November 2016.

_ , Gauti B Eggertsson, and Bulat Gafarov, "Time Consistency and Duration of Government Debt: A Model of Quantitative Easing," *The Review of Economic Studies*, 09 2022, *90* (4), 1759–1799.

Breedon, Francis, Jagjit S. Chadha, and Alex Waters, "The financial market impact of UK quantitative easing," *Oxford Review of Economic Policy*, WINTER 2012, *28* (4), 702–728.

Cahill, Michael E., Stefania D'Amico, Canlin Li, and John S. Sears, "Duration risk versus local supply channel in Treasury yields: evidence from the Federal Reserve's asset purchase announcements," Finance and Economics Discussion Series 2013-35, Board of Governors of the Federal Reserve System (U.S.) 2013.

Campbell, Jeffrey R., Charles L. Evans, Jonas D.M. Fisher, and Alejandro Justiniano, "Macroeconomic Effects of Federal Reserve Forward Guidance," *Brookings Papers on Economic Activity*, 2012, *2012* (2), 1–54.

Chen, Han, Vasco Cúrdia, and Andrea Ferrero, "The Macroeconomic Effects of Large?scale Asset Purchase Programmes," *Economic Journal*, November 2012, *122* (564), 289–315.

Christensen, Jens and Glenn Rudebusch, "The Response of Interest Rates to US and UK Quantitative Easing," *Economic Journal*, 2012, *122* (564), F385–F414.

Cui, Wei and Vincent Sterk, "Quantitative easing with heterogeneous agents," *Journal of Monetary Economics*, 2021, *123* (C), 68–90.

Cúrdia, Vasco and Michael Woodford, "The central-bank balance sheet as an instrument of monetarypolicy," *Journal of Monetary Economics*, January 2011, *58* (1), 54–79.

Debortoli, Davide and Jordi Galí, "Monetary policy with heterogeneous agents: Insights from TANK models," Economics Working Papers 1686, Department of Economics and Business, Universitat Pompeu Fabra September 2017.

D'Amico, Stefania and Thomas B. King, "Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply," *Journal of Financial Economics*, 2013, *108* (2), 425–448.

Eser, Fabian and Bernd Schwaab, "Evaluating the impact of unconventional monetary policy measures: Empirical evidence from the ECB's Securities Markets Programme," *Journal of Financial Economics*, 2016, *119* (1), 147–167.

Eyster, Erik, Matthew Rabin, and Dimitri Vayanos, "Financial Markets Where Traders Neglect the Informational Content of Prices," *The Journal of Finance*, 2019, *74* (1), 371–399.

Gagnon, Joseph E., Matthew Raskin, Julie Remache, and Brian P. Sack, "Large-scale asset purchases by the Federal Reserve: did they work?," *Economic Policy Review*, 2011, *17* (May), 41–59.

Gertler, Mark and Peter Karadi, "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, 2015, *7* (1), 44–76.

Gilchrist, Simon, David López-Salido, and Egon Zakrajšek, "Monetary Policy and Real Borrowing Costs at the Zero Lower Bound," *American Economic Journal: Macroeconomics*, January 2015, *7* (1), 77–109.

Gornemann, Nils, Keith Kuester, and Makoto Nakajima, "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy," International Finance Discussion Papers 1167, Board of Governors of the Federal Reserve System (U.S.) May 2016.

Greenwood, Robin and Dimitri Vayanos, "Bond Supply and Excess Bond Returns," *Review of Financial Studies*, 2014, *27* (3), 663–713.

Hagedorn, Marcus, Jinfeng Luo, Iourii Manovskii, and Kurt Mitman, "Forward guidance," *Journal of Monetary Economics*, 2019, *102* (C), 1–23.

Hamilton, James D. and Jing Cynthia Wu, "The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment," *Journal of Money, Credit and Banking*, February 2012, *44* (s1), 3–46.

Harrison, Richard, "Optimal quantitative easing," Bank of England working papers 678, Bank of England September 2017.

Hellwig, Christian, Arijit Mukherji, and Aleh Tsyvinski, "Self-Fulfilling Currency Crises: The Role of Interest Rates," *The American Economic Review*, 2006, *96* (5), 1769–1787.

Iovino, Luigi and Dmitriy Sergeyev, "Central Bank Balance Sheet Policies Without Rational Expectations," *The Review of Economic Studies*, 02 2023, *90* (6), 3119–3152.

Jeanne, Olivier and Lars E. O. Svensson, "Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank," *American Economic Review*, March 2007, *97* (1), 474–490.

Joyce, Michael, David Miles, Andrew Scott, and Dimitri Vayanos, "Quantitative Easing and Unconventional Monetary Policy – an Introduction," *Economic Journal*, November 2012, *122* (564), 271–288.

Judd, Kenneth L., "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 1985, *28* (1), 59–83.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante, "Monetary Policy According to HANK," *American Economic Review*, March 2018, *108* (3), 697–743.

Kim, Kyungmin, Thomas Laubach, and Min Wei, "Macroeconomic Effects of Large-Scale Asset Purchases: New Evidence," Finance and Economics Discussion Series 2020-047, Board of Governors of the Federal Reserve System (U.S.) June 2020.

King, Thomas B., "A Portfolio-Balance Approach to the Nominal Term Structure," Working Paper Series WP-2013-18, Federal Reserve Bank of Chicago November 2013.

_ , "Expectation and duration at the effective lower bound," *Journal of Financial Economics*, 2019, *134* (3), 736–760.

Koijen, Ralph S. J., François Koulischer, Benoît Nguyen, and Motohiro Yogo, "Euro-Area Quantitative Easing and Portfolio Rebalancing," *American Economic Review*, May 2017, *107* (5), 621–27.

Koijen, Ralph S.J., François Koulischer, Benoît Nguyen, and Motohiro Yogo, "Inspecting the mechanism of quantitative easing in the euro area," *Journal of Financial Economics*, 2021, *140* (1), 1–20.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," *Brookings Papers on Economic Activity*, 2011, *42* (2 (Fall)), 215–287.

_, Stefan Nagel, and Annette Vissing-Jorgensen, "ECB Policies Involving Government Bond Purchases: Impacts and Channels," CEPR Discussion Papers 12399, C.E.P.R. Discussion Papers October 2017.

Li, Canlin and Min Wei, "Term Structure Modeling with Supply Factors and the Federal Reserve's Large-Scale Asset Purchase Progarms," *International Journal of Central Banking*, March 2013, *9* (1), 3–39.

Luetticke, Ralph, "Transmission of Monetary Policy with Heterogeneity in Household Portfolios," Discussion Papers 1819, Centre for Macroeconomics (CFM) June 2018.

McLaren, Nick, Ryan N. Banerjee, and David Latto, "Using Changes in Auction Maturity Sectors to Help Identify the Impact of QE on Gilt Yields," *The Economic Journal*, 05 2014, *124* (576), 453–479.

Mussa, Michael, *The Role of Official Intervention*, Group of Thirty New York, 1981.

Negro, Marco Del, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki, "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities," *American Economic Review*, March 2017, *107* (3), 824–57.

Ravn, Morten O and Vincent Sterk, "Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach," *Journal of the European Economic Association*, 2021, *19* (2), 1162–1202.

Rogers, John H., Chiara Scotti, and Jonathan H. Wright, "Unconventional Monetary Policy and International Risk Premia," *Journal of Money, Credit and Banking*, December 2018, *50* (8), 1827–1850.

Sims, Eric and Jing Cynthia Wu, "Evaluating Central Banks' tool kit: Past, present, and future," *Journal of Monetary Economics*, 2021, *118*, 135–160.

Vayanos, Dimitri and Jean-Luc Vila, "A Preferred-Habitat Model of the Term Structure of Interest Rates," *Econometrica*, January 2021, *89* (1), 77–112.

Wallace, Neil, "A Hybrid Fiat-Commodity Monetary System," *Journal of Economic Theory*, 1981, *25* (3), 421–430.

Wen, Yi, "When and how to exit quantitative easing?," *Federal Reserve Bank of St. Louis Review*, 2014, *96* (3), 243–265.