

Self-Fulfilling Debt Crises, Fiscal Policy and Investment*

Carlo Galli^a

^a*Department of Economics, Sciences Po, Paris, France.*

Abstract

This paper studies the circular relationship between sovereign credit risk, government fiscal and debt policy, and output. I consider a sovereign default model with fiscal policy and private capital accumulation. I show that, when fiscal policy responds to borrowing conditions in the sovereign debt market, multiple equilibria exist where the expectations of lenders are self-fulfilling. In the bad equilibrium, pessimistic beliefs make sovereign debt costly. The government substitutes borrowing with taxation, which depresses private investment and future output, increases default probabilities and verifies lenders' beliefs. This result is reminiscent of the European debt crisis of 2010-12: while recessionary, fiscal austerity may be the government best response to excessive borrowing costs during a confidence crisis. The extent of this type of crises can be reduced by the intervention of an external official lender committing to provide liquidity, or by policies aimed at sustaining private investment.

Keywords: Self-fulfilling debt crises, sovereign default, multiple equilibria, fiscal austerity

JEL: E44, E62, F34, G12

1. Introduction

The European debt crisis of 2010-12 raised, in both academic and policy circles, two important issues: one relates to the possibility of self-fulfilling debt crises in sovereign debt markets, the other to the effectiveness of austerity policies. Pessimistic investors' beliefs on government solvency have often been cited to explain, at least partly, the spike observed in government bond spreads starting from 2010, and their subsequent reduction following interventions by the European Central Bank. Austerity policies have sparked a heated debate during the crisis, when fiscal consolidation measures were adopted by southern European countries as a response to the turbulence in sovereign debt markets. Some considered these policies necessary to reduce debt levels and decrease exposure to debt market fluctuations; others argued that their effects were largely contractionary and worsened the debt crisis.

These two issues are related by the existence of a negative feedback loop between bond spreads, government fiscal and debt policy, and economic activity. Bond spreads can have a significant impact on policy, because they affect the cost of borrowing and in turn the policy mix behind the government funding strategy. There is ample descriptive evidence that the rise in sovereign spreads observed during the European debt

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Email address: carlo.galli@sciencespo.fr (Carlo Galli)

crisis was a concern for policymakers, and in many occasions the motivation for austerity measures¹ that proved to adversely impact consumption, investment and output. The relationship between government bond prices and economic activity closes the circle, as default incentives tend to be increasing in debt to GDP ratios, being stronger during recessions and when debt stocks are large.

This paper studies in detail the circular relationship between spreads, policy and output, providing a tractable framework to characterise under what conditions there may exist multiple equilibria where the beliefs of sovereign debt market participants are self-fulfilling. In the model, a confidence crisis makes it costlier for the government to obtain external funding, forcing it to increase domestic taxation instead. Higher taxes depress private investment and in turn future output, increasing future default probabilities and ultimately verifying lenders' pessimistic beliefs, resulting in an equilibrium that is bad for the government. If instead borrowing conditions are favourable, the government can borrow more cheaply and tax less, so investment is high and default probabilities are in turn low. The bad equilibrium illustrates a debt crisis episode similar to that experienced by some Southern European countries during the European debt crisis: fiscal consolidation is the government best response to excessive borrowing costs, but it has real, contractionary effects and is accompanied by low domestic welfare.

I propose a simple two-period model building on the tradition of [Eaton and Gersovitz \(1981\)](#) and the subsequent quantitative work of [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#). I consider a risk-averse, benevolent government that trades defaultable debt with a continuum of foreign risk-neutral investors, and taxes domestic households. Households accumulate capital, produce according to a concave production technology, pay taxes to the government and consume. The government chooses debt, tax and default policy to maximise the utility of domestic households, who suffer a random production loss in case of default. I assume that the government cannot commit to future actions, and that tax policy and private investment are chosen after the debt auction. This assumption implies that the government adjusts to external borrowing conditions with debt as well as fiscal policy, and the latter affects the private sector consumption-saving decision. Private investment determines future output and, in turn, future default incentives, which affect debt prices via lenders' expectations. The model also features an externality in aggregate investment, because households are atomistic and fail to internalise the effect of future capital on default probabilities, bond prices and taxes. The circular relationship between government bond prices, fiscal policy and private investment creates the possibility of multiple equilibria driven by self-fulfilling expectations on the side of foreign investors.

Following the quantitative literature, I assume that the government moves first in the debt issuance game, choosing debt at maturity (i.e. fixing its future repayment obligations). Lenders then bid a price, being willing to lend to the government as long as they make zero profits in expectation. For some levels of debt issuance, there exist multiple debt prices that satisfy such zero profit condition. This coordination problem among lenders is the key mechanism behind the existence of multiple equilibria in the model. It relies on the effect that debt prices have, via government taxation, on household wealth in the first period, which in turn affects investment and government default incentives. I adopt a selection criterion that rules out unstable outcomes, and determines the prices on which creditors coordinate and the terms at which the government can borrow. I then characterise with two propositions the optimal policy of the government as a function of the debt price schedule it faces and its initial endowment, and show with a numerical example the existence of multiple equilibria that depend on lenders' self-fulfilling beliefs.

A natural question to ask is what policy can do to prevent a debt crisis and increase households' welfare. In Section 4 I consider how a large, benevolent third party lender (such as the IMF for emerging market borrowers or the ESM for Eurozone countries) can help address lenders' coordination failure by providing liquidity to the government. When the IMF commits to buy a sufficiently large share of debt issuance, its intervention can prevent a debt crisis. I show that interventions with a pari-passu status are more effective than those with a senior, or preferred, creditor status. Finally, I show that domestic or external policies

¹See for example the Monti government press release on the "Salva Italia" decree (December 4, 2011, a transcript can be found at https://www.repubblica.it/economia/2011/12/04/news/la_manovra_di_monti_comunicato_ufficiale-26095635 (last accessed on 24/03/2021)), or the Spanish parliament debate regarding the austerity plan put forward by the Zapatero government (February 17, 2010, https://www.congreso.es/public_oficiales/L9/CONG/DS/PL/PL_140.PDF (last accessed on 24/03/2021)).

aimed at sustaining investment, such as investment subsidies, can also prevent lenders' coordination failure, as well as correct investment externalities when they increase the government commitment power.

I choose a specification of the model that makes it tractable and allows to present the main mechanisms in a transparent way. The results of the paper however are general, in the sense that the feedback loop linking bond spreads, output and default incentives can also be modelled in other ways. The necessary ingredient is that spreads have real effects that affect future default incentives. As in this paper, this transmission can be intermediated by policy: multiple equilibria are possible as long as the government undertakes some domestic policy action that (i) responds to current borrowing conditions, and (ii) affects its future repayment incentives, either directly or indirectly through the private sector. Examples of such policy are government reform effort, productive government spending, or taxation: actions that are costly today but increase the likelihood of higher growth tomorrow, or vice versa. Another possibility is that spreads affect real activity directly, for example through the banking sector. I discuss this mechanism more in detail in Section 5: while I do not explore it formally in this paper, I consider it a force that is complementary to the one analysed here.

Related Literature. This paper mainly relates to two strands of the literature on sovereign debt and default. The first concerns debt crises and multiplicity in sovereign default models. As shown by [Auclert and Rognlie \(2016\)](#), the sovereign default framework in the tradition of [Eaton and Gersovitz \(1981\)](#), most common in the quantitative literature, features a unique equilibrium if debt is short-term. To analyse the role of beliefs, the literature on multiple equilibria relies on modifications of this framework along several dimensions. [Calvo \(1988\)](#) and subsequent work by [Lorenzoni and Werning \(2019\)](#) (LW henceforth) and [Ayres et al. \(2018\)](#) assume a different structure for the government debt auction: the government fixes current auction revenues, and future repayment obligations depend on debt prices, which are taken as given. In this framework, high interest rates imply high future debt, which makes default probabilities high and in turn justifies the high interest rates. Other papers, from the workhorse model of [Cole and Kehoe \(2000\)](#) to more recent work by [Aguiar et al. \(2016\)](#) and [Conesa and Kehoe \(2017\)](#), consider rollover risk by adopting a different timing assumption, whereby the government can issue new debt before deciding whether to default on both new and pre-existing debt. [Aguiar and Amador \(2020\)](#) and [Stangebye \(2020\)](#) show that a different, dynamic type of equilibrium multiplicity may exist if the Eaton-Gersovitz model is extended to allow for long-term debt.² [Corsetti and Maeng \(2020\)](#) consider both fast and slow crises to analyse how debt maturity interacts with the possibility of either crisis. [Aguiar et al. \(2015\)](#), [Corsetti and Dedola \(2016\)](#) and [Bassetto and Galli \(2019\)](#) analyse the interplay between self-fulfilling beliefs and inflation, when debt is denominated in local currency. [Bocola and Dovis \(2019\)](#) evaluate quantitatively the contribution of fundamentals and beliefs in explaining the behaviour of government bond spreads.

An important feature common to all this literature is that it focuses on the interaction between government debt policy and bond spreads: default incentives depend on debt/GDP ratios, but output is assumed exogenous.³ However, default risk can have real effects on economic activity, either directly or through government policy. I contribute to the literature by showing that the way in which fiscal policy responds to a confidence crisis may well be the reason why such crisis is indeed self-confirming. In other words, I show one way in which the denominator, rather than the numerator, of the debt/GDP ratio can be the channel through which lenders' expectations are self-fulfilling.

The second stream of literature relevant for this paper is that on sovereign default models with dynamic policy and endogenous output.⁴ [Gordon and Guerron-Quintana \(2018\)](#) and [Bai and Zhang \(2012\)](#) study quantitative models of default risk and capital accumulation that are similar to the framework presented in this paper. The crucial difference lies in their assumption that domestic policy is contractible, so debt prices do not affect investment but are rather a function of it. [Müller et al. \(2019\)](#) model domestic policy as effort

²[Cui and Kaas \(2021\)](#) consider a related type of dynamic multiplicity within the context of corporate debt and default.

³[Cole and Kehoe \(2000\)](#) do consider a model with capital and production, but there is no interaction between government fiscal policy and households' investment decisions.

⁴[Arellano and Bai \(2017\)](#) and [Balke and Ravn \(2016\)](#) also analyse sovereign default and fiscal policy in a model with endogenous output, but assume that both policy and production are static.

to undertake structural reforms, which is assumed to have a separable cost and thus does not interact with lenders' beliefs in a way that creates the possibility of multiple equilibria. Broner et al. (2014) consider a model with capital and explore the possibility of belief-driven equilibria; in their model multiplicity is driven by a crowding-out effect of government debt on capital, and by its interplay with creditor discrimination. Their mechanism is different and complementary to that analysed in this paper.

Closest to my work is Detragiache (1996). She sketches a general framework where the government is facing a planner's problem in which policy effort is non-contractible, has non-separable costs and positively affects future repayment probabilities. She observes that multiple equilibria are possible when lenders' coordination failure reduces lending and forces the government to provide less effort. My paper extends this intuition in a number of ways: first, I characterise the equilibrium policy fully, focusing exclusively on stable equilibria; second, I show how the equilibrium can be decentralised, and relate it to the discussion on "fiscal austerity" central to the European debt crisis; finally, I consider how different policy interventions can prevent lenders' coordination failure and improve upon the equilibrium outcomes.

This paper also relates to the literature on debt overhang and investment, because of the negative effect that default probabilities have on investment incentives. Krugman (1988) and Sachs (1989) show that, when debt levels are high and taken as given, investment is discouraged because most of the return accrues to creditors. In Lamont (1995), corporate debt overhang can create complementarities in investment that generate multiple equilibria driven by expectations, in a way similar to the coordination problem among households that is present in this paper. In more recent work, Aguiar et al. (2009) show that limited commitment on the side of the government leads to under-investment in bad times and when debt is large, as is true for equilibrium policy in this paper when creditors' expectations severely constrain borrowing.

The remainder of the paper proceeds as follows: Section 2 presents the two-period model; Section 3 illustrates the key mechanisms at play with a numerical example and characterises the equilibrium; Section 4 discusses policy interventions; Section 5 discusses some extensions to the model, as well as the main assumptions behind the results; Section 6 concludes. Appendix A presents a simplified, deterministic version of the two-period model that allows to derive closed-form results, and Appendix B contains proofs and derivations.

2. Two-Period Model

I consider a small open economy with a continuum of measure one of identical households and a government. Time is discrete and there are two periods, $t = 0, 1$.

The government is benevolent and wishes to maximise households' utility. It starts period $t = 0$ with a stock of debt due equal to B_0 . It finances the repayment of such debt by borrowing new one-period debt B_1 at a price of q_0 from international lenders, and collecting lump-sum taxes T_0 . The government cannot commit to repay debt at $t = 1$, so it will default whenever it finds convenient to do so. In case of repayment, it collects lump-sum taxes T_1 from households. Default causes a random, proportional output loss z_1 but it writes debt off completely, so the government does not need to tax households.⁵ The random output loss z_1 is assumed to be distributed according to a cumulative distribution function $G(z_1)$ with support $Z \subseteq [0, 1]$. Following LW I assume that initial debt B_0 cannot be defaulted upon in period $t = 0$. The budget constraints of the government are given by

$$\begin{aligned} B_0 &= T_0 + q_0 B_1 \\ (1 - \delta_1) B_1 &= T_1 \end{aligned}$$

where δ_1 is a binary variable that takes the value of 1 if the government defaults, and 0 otherwise. Henceforth I will mention debt and tax policy interchangeably since either one pins down the other, conditional on debt price q_0 and initial debt level B_0 . In Section 5 I argue that the results of the model are robust to the

⁵The reduced form assumption that default has a direct output cost is made for tractability. It is analogous to assuming that the productivity of capital is random and default causes the loss of a fixed share of output. The interpretation of this assumption is linked to a large area of the literature where defaults are shown to cause losses in output due, for example, to disruptions in access to international trade or in the domestic banking sector.

alternative assumption that the government taxes production (or equivalently consumption) proportionally rather than in a lump-sum way. I choose the specification with lump-sum taxation because it allows for a clearer characterisation of equilibrium policy.

Households have preferences represented by the utility function

$$u(c_0) + \beta \mathbb{E}_0 u(c_1)$$

over individual consumption levels $\{c_0, c_1\}$, where $u(c_t) := c_t^{1-\gamma}/(1-\gamma)$ and $\gamma > 0$. They produce output using individual capital k_t according to a concave production function $f(k_t) := k_t^\alpha$ with $\alpha \in (0, 1)$, and pay lump-sum taxes T_t to the government. Households start with an initial stock of capital equal to k_0 and can only save through capital. For simplicity, I assume that capital fully depreciates over time, and that households produce using a backyard technology, the output of which they consume directly.⁶ The household budget constraints are given by

$$\begin{aligned} c_0 &= f(k_0) - k_1 - T_0 \\ c_1 &= f(k_1) - T_1 \end{aligned}$$

where initial capital k_0 is given.

We can now examine the default decision of the government. Let us plug the government budget constraint at $t = 1$ into that of the households, and denote aggregate capital in period t with K_t . The optimal default decision solves

$$\max_{\delta_1} (1 - \delta_1) u\left(f(K_1) - B_1\right) + \delta_1 u\left(f(K_1)(1 - z_1)\right).$$

It follows that the government defaults on its debt obligations if and only if the output cost of defaulting is smaller than a threshold equal to the ratio of debt to GDP, which mathematically is equivalent to

$$z_1 < \widehat{z}_1(K_1, B_1) := \frac{B_1}{f(K_1)}. \quad (1)$$

When the government is indifferent, I assume that it chooses repayment. Importantly, default incentives are decreasing in output and increasing in debt, as is commonly assumed in the sovereign default literature. Henceforth I will omit the arguments of \widehat{z}_1 in order to lighten notation.

There are two important aspects of the model that are related to timing and government commitment. First, I assume that households take as given the quantity of debt issued and its price, and in turn tax policy. They then form expectations about default in period $t = 1$ conditional on government debt and tax policy. It follows that private sector behaviour during the time between debt issuance and maturity affects the evolution of GDP and in turn the government default incentives at maturity. Second, government tax policy is assumed to adjust to the outcome of the government debt auction, in order to ensure that the government budget constraint at $t = 0$ is satisfied.⁷ These aspects have two important implications. First, foreign lenders who price government debt anticipate the response of the private sector to the outcome of the debt auction, and in turn to taxes. Second, there is an externality due to the fact that households do not internalise the effect that investment has on future default incentives and in turn current debt prices.⁸

I now examine the household capital investment decision. As households are identical and have the same initial stock of capital, $k_0 = K_0$. Let us replace first-period taxes T_0 with government net lending $B_0 - q_0 B_1$

⁶Assuming instead that production is carried out by a representative firm that rents capital and hires labour (supplied inelastically) from households would deliver the same results.

⁷This apparently obvious assumption is actually related to the important distinction, carefully explained in [Bassetto \(2005\)](#), between government commitment to unconditional actions or to strategies. I elaborate more on this point in Subsection 4.4.

⁸Other work on sovereign debt and investment, such as [Bai and Zhang \(2012\)](#), [Gordon and Gueron-Quintana \(2018\)](#) and [Esquivel \(2020\)](#), instead assume that investment is contractible and chosen by the government, who internalises the impact on debt prices. This implies that capital is an argument of the price function for debt, which eliminates the scope for multiple belief-driven equilibria in the sovereign debt market.

inside the household budget constraint, and denote initial aggregate wealth with $W_0 := f(K_0) - B_0$. W_0 will be the relevant state variable for both the household and the government problem. Optimal individual investment solves

$$\begin{aligned} \max_{k_1} V_0(k_1; W_0, q_0, B_1, K_1) := \max_{k_1} \left\{ u(W_0 + q_0 B_1 - k_1) + \beta \int_{\hat{z}_1} u(f(k_1) - B_1) dG(z_1) \right. \\ \left. + \beta \int^{\hat{z}_1} u(f(K_1)(1 - z_1)) dG(z_1) \right\} \end{aligned} \quad (2)$$

and is thus a function of initial wealth W_0 , government debt policy B_1 , debt price q_0 and aggregate investment K_1 .⁹ Solving (2) and imposing the symmetric equilibrium condition $k_1 = K_1$ yields the aggregate private sector investment response function

$$\mathcal{K}(W_0, q_0, B_1) := \{K_1 : K_1 \in \arg \max_{k_1} V_0(k_1; W_0, q_0, B_1, K_1)\}. \quad (3)$$

In principle, there could be multiple solutions to the fixed point problem of equation (2) due to complementarities in household investment: a coordination failure among households is possible, where the belief of low aggregate investment and high default likelihood discourages individual investment and is self-confirming. Note that this coordination problem among households is separate from, and independent of, the coordination problem among lenders. Numerical explorations of the model suggest that this multiplicity has negligible implications for the purpose of characterising the equilibrium. To keep the exposition simple, I decide to ignore this type of coordination failure by assuming that, when there exist multiple solutions to (2), households always coordinate on the highest investment level. Henceforth I will thus consider $\mathcal{K}(W_0, q_0, B_1)$ as a single-valued function.

Foreign lenders are risk-neutral and perfectly competitive. There is a continuum of them, of measure large enough that their aggregate lending capacity is always sufficient to buy all of the government debt issuance. Lenders are thus willing to buy any amount of debt as long as they make zero profits in expectation. The assumption that lenders are atomistic is crucial for the existence of complementarities in the debt issuance game. In case of default, lenders recover an amount of funds equal to a fraction η of the output lost by the government, and these funds are shared equally among all bondholders. This implies that the recovery value *per bond* held is given by

$$\eta \frac{z_1 f(K_1)}{B_1} \quad (4)$$

which, by definition, is also equal to $\eta \hat{z}_1 / z_1$. An important implication of this assumption is that the recovery value is decreasing in the stock of debt; as we will see in Subsection 4.3, this feature allows for a non trivial analysis of the role of creditors' seniority. Finally, it is assumed that lenders' discount factor is given by $1/R$ and that the government issues discount bonds, all of which implies that the risk-free price of debt is equal to $1/R$.

A further assumption is needed regarding the specific timing of the government debt auction. Following most of the quantitative sovereign default literature, I adopt the timing structure of [Eaton and Gersovitz \(1981\)](#), whereby the government moves first and chooses the quantity of debt it wishes to issue, and then lenders bid and determine the issuance price. It is well-known that, when output is exogenous and debt is short-term, this assumption generally leads to equilibrium uniqueness.¹⁰ A key point of this paper is that, if fiscal policy has real, dynamic effects on output, then multiple equilibria can arise even in the Eaton-Gersovitz timing.¹¹ Lenders are willing to buy debt at any zero profit price q_0 that satisfies the equation

$$q_0 = \frac{1}{R} \left[1 - G(\hat{z}_1) + \int^{\hat{z}_1} \eta \frac{z_1}{z_1} dG(z_1) \right], \quad (5)$$

⁹Recall that the default cutoff \hat{z}_1 is a function of aggregate, not individual, capital.

¹⁰For a discussion in both finite and infinite horizon settings, see LW, [Ayres et al. \(2018\)](#), [Aguar and Amador \(2020\)](#), [Auclert and Rognlie \(2016\)](#) and [Passadore and Xandri \(2019\)](#).

¹¹Section 5 discusses timing in the context of the existing literature more in detail.

where \hat{z}_1 is a function of B_1 and K_1 , and $K_1 = \mathcal{K}(W_0, q_0, B_1)$. Repayment probabilities depend on debt as well as capital. Private sector capital investment depends on debt auction revenues through a wealth effect on households in period $t = 0$: in order to repay initial debt B_0 , the government must finance with domestic taxes what it does not raise in sovereign debt markets. Because of this, there may exist multiple solutions to equation (5) for some (W_0, B_1) pairs.¹² This is the core source of multiple equilibria of the model, and will be examined more in detail later. Henceforth, I will refer to the right-hand side of equation (5) as the “expected value of debt”, and to any solution of equation (5) for q_0 as a zero profit price.

I now define the notion of equilibrium, focusing on symmetric equilibria where all households and lenders follow the same actions.

Definition 1. *A competitive equilibrium is a collection of government debt and default choices (B_1, δ_1) , households’ investment choice K_1 and a debt price function $\mathcal{Q}(W_0, B_1)$ such that, given initial wealth W_0 ,*

1. *households choose investment to maximise their expected utility, given government policies and debt prices;*
2. *the debt price function $\mathcal{Q}(W_0, B_1)$ satisfies creditors’ zero-profit condition for all debt levels $B_1 \in \mathbb{R}$;*
3. *government policies maximise households’ expected utility, subject to the households’ investment response and the debt price function.*

I restrict the analysis to the set of initial wealth levels (i.e. initial (K_0, B_0) pairs) such that the household budget set allows for positive consumption and investment levels.

Combining conditions (1), (2) and (5) we can focus on the government problem of choosing debt in period $t = 0$ to maximise households’ utility, subject to its optimal default policy in period $t = 1$, creditors’ zero-profit condition and households’ investment response:

$$\begin{aligned}
& \max_{C_0, C_1^R, C_1^D, B_1, q_0, K_1} u(C_0) + \beta \left[u(C_1^R)(1 - G(\hat{z}_1)) + \int^{\hat{z}_1} u(C_1^D) dG(Z_1) \right] \\
& \text{s.t. } C_0 = W_0 + q_0 B_1 - K_1 \\
& \quad C_1^R = f(K_1) - B_1 \\
& \quad C_1^D = f(K_1)(1 - z_1) \\
& \quad q_0 = \mathcal{Q}(W_0, B_1) \\
& \quad K_1 = \mathcal{K}(W_0, q_0, B_1) \\
& \quad W_0 \text{ given}
\end{aligned} \tag{6}$$

where $\mathcal{K}(W_0, q_0, B_1)$ and $\mathcal{Q}(W_0, B_1)$ are single-valued functions that the government takes as given. \mathcal{Q} is selected¹³ from the correspondence that maps (W_0, B_1) pairs into the set of zero profit prices that satisfy (5).

3. Multiplicity and Equilibrium Policy

This section presents equilibrium policy and highlights the key mechanisms of the model laid out in the previous section. I derive optimality conditions and characterise the general features of equilibrium policy, while presenting a numerical example that shows the existence of multiple equilibria and their properties. In the appendix, I present a version of the model that admits a closed-form, almost complete characterisation of the equilibrium. The choice to relegate such model to the appendix is dictated by the fact that a closed-form solution can only be obtained by making a number of assumptions that eliminate or complicate interesting aspects of the model presented in the main text.

¹²Mathematically, multiple solutions to equation (5) may exist because q_0 shows up linearly on the left-hand side and non-linearly on the right-hand side, inside $\hat{z}_1(\mathcal{K}(W_0, q_0, B_1), B_1)$.

¹³See Section 3.3 for details on how this selection is made.

The analysis proceeds in three steps. First, I examine the private sector investment response. Second, I show that there may exist multiple zero profit prices consistent with a given level of debt issuance. I show under what conditions this happens, and I adopt a selection criterion that determines the prices on which lenders coordinate, considering only stable solutions. Third, I analyse the government optimal policy, and show that multiple equilibria exist where policy depends on the debt price schedule faced by the government.

In the parametric example I use the following parameters: capital share of output $\alpha = 0.4$, risk-aversion parameter $\gamma = 1$ (log utility), households' discount factor $\beta = 0.9$, lenders' opportunity cost of capital $R = 1.05$, recovery parameter $\eta = 0.9$, default output cost normally distributed with mean 0.5, standard deviation 0.035 and support $Z = [0, 1]$. When not specified, figures use the same variable values of the previous figures.

3.1. Private Sector Investment Response

I start by considering households' investment decision conditional on government debt issuance B_1 and debt price q_0 (or, equivalently, tax policy T_0). Households take future default and tax policy as given because they are atomistic, so they do not internalise the effect of their choice on aggregate investment. The individual best response $k_1^*(K_1)$ to aggregate investment K_1 solves the household investment problem in (2) and is given by the following first-order condition

$$\frac{1}{W_0 + q_0 B_1 - k_1} = \beta f'(k_1) \left[\frac{1 - G(\hat{z}_1)}{f(k_1) - B_1} + \frac{G(\hat{z}_1)}{f(k_1)} \right]. \quad (7)$$

Note that the assumption of log utility simplifies the expression because the effect of default cost $(1 - z_1)$ on the marginal product of capital and on default consumption cancel out exactly.

Equation (7) highlights a form of debt overhang mechanism. The marginal benefit of investment is lower when debt/GDP is high and default is likely, because it implies a loss in productivity. This creates complementarities in household investment, because individual investment incentives are stronger, the larger is aggregate investment, and the lower are default probabilities. These complementarities have two main implications. First, in principle there may exist multiple fixed point solutions $k_1^*(K_1) = K_1$ to equation (7) for a given (W_0, q_0, B_1) triplet. For simplicity, here I focus on equilibria where households always coordinate on the highest investment level. Second, aggregate investment responds to debt prices in a nonlinear way, which strengthens the mechanism behind the existence of multiple zero profit prices. The next subsection analyses this property in more detail.

3.2. Debt Price Schedules

I now examine creditors' zero profit condition: for a given level of debt issuance B_1 and initial wealth W_0 , I consider whether there exist multiple zero profit prices that solve equation (5). Earlier, I analysed private sector investment taking government policy (debt and taxes) *and* debt prices as given. Now I go backwards in the order of play within the first period, and I consider how a change in the price of debt affects private investment through taxation. This approach is consistent with the timing of the government debt auction: lenders bid a price after the government has chosen how much debt to issue, anticipating the effect of debt auction revenues on government tax policy, and in turn on households' wealth, in period $t = 0$.

Changes in the price of debt q_0 affect government taxation T_0 , aggregate investment K_1 , the default threshold $\hat{z}_1(K_1, B_1)$ and ultimately the expected value of debt given by equation (5) via default probabilities and expected recovery upon default. Figure 1 plots the response of two crucial variables in this mechanism: private sector aggregate investment $\mathcal{K}(W_0, q_0, B_1)$, on the right panel, and the expected value of debt for lenders on the left panel. Both variables are plotted as a function of the price of debt q_0 , for a given initial wealth level W_0 and three different levels of debt issuance B_1 .

Let us start by considering the left panel, which shows the expected value of debt for lenders. Zero profit prices are represented by round markers and correspond to the points where the curves intersect the 45-degree dotted line. When the expected value of debt is a sufficiently nonlinear function of the price of debt q_0 , there may exist multiple zero profit prices for some debt levels. In the example of the figure, this

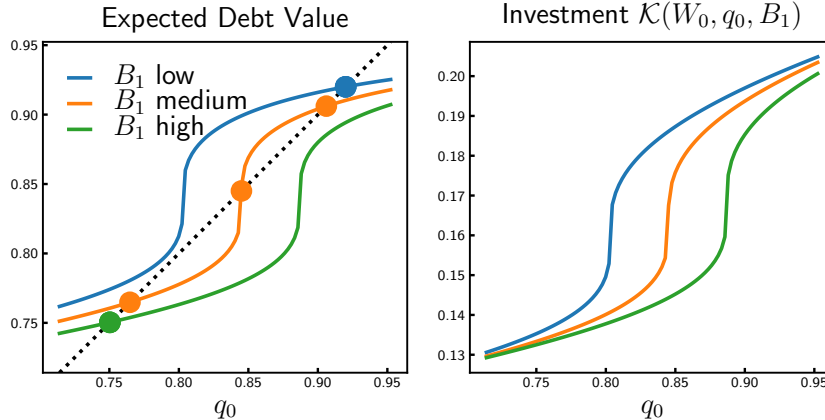


Figure 1: The effect of debt prices on investment and debt value. Notes: the figure plots expected debt value and zero profit prices (left panel) and capital investment (right panel) as a function of debt price q_0 , given initial wealth W_0 and different levels of debt issuance B_1 . The expected debt value is defined as the right-hand side of equation (5); a zero profit price is any solution to such equation. For this example, $W_0 = 0.3$ and $B_1 = \{0.25, 0.2525, 0.255\}$.

happens when B_1 has an intermediate value (orange curve). Multiplicity is driven by two mechanisms: the response of default probabilities to aggregate investment, and the response of aggregate investment to debt prices.

The response of default probabilities to aggregate investment depends on the shape of the default cost distribution. The similarity between the curves in the two panels of the figure may suggest that all of the nonlinearity in the expected debt value is driven by the nonlinearity in the investment response to q_0 . This is however not necessarily the case: in Appendix A I present a different specification of the model where tax externalities at $t = 1$ make aggregate investment a simple linear function of debt prices. That allows to isolate the effect of the default cost distribution in generating multiple zero profit prices: I show that multiplicity is present if the variance of the distribution of z_1 is sufficiently low.

The response of aggregate investment to debt prices is driven by a wealth effect on household investment decisions at $t = 0$, and by the interaction between investment incentives and default probabilities at $t = 1$. The right panel of Figure 1 shows the aggregate investment response as determined by equation (7) after imposing the equilibrium condition $k_1 = K_1$. First, investment is strictly increasing in q_0 , because higher debt prices have a positive wealth effect on households in period $t = 0$: higher debt revenues for the government allow for lower taxes and higher initial net worth for households, which increases their willingness to save. Second, the figure shows that \mathcal{K} has a clear non-linearity, an almost vertical jump, in q_0 . This feature is not essential in generating multiple zero profit prices, but it strengthens the mechanism behind their existence. The non-linearity is due to the fact that investment incentives are increasing in repayment probabilities, since a default causes a drop in the marginal product of capital and the marginal benefit of saving. Mathematically, the household objective function is not strictly concave in investment. The convexity appears at points where a small change in K_1 has a large effect on default probabilities, i.e. when the default cutoff lies in regions of the distribution of z_1 with a high probability density. In fact, the objective function may display two local maxima. One maximum corresponds to situations where q_0 is low, taxes are high, investment is low, default is likely, and high default probabilities indeed provide little incentive to invest. Vice versa, the other maximum is characterised by high q_0 , low taxes, higher investment and repayment probabilities, with the latter reinforcing the incentive to invest. The sharp increase in investment coincides with the point where the global maximum in the investment objective function switches from the local maximum with high default probabilities to that with low ones.

Multiplicity is thus driven by the combination of non-linearities in the response of default probabilities to investment on one hand, and in the response of investment to debt prices on the other hand. When debt

belongs to an intermediate region (B_1 medium in Figure 1), the joint effect of these forces determines the existence of multiple zero-profit prices. When instead issued debt is too large or small, changes in debt prices do not affect investment and the default cutoff \hat{z}_1 enough to allow for the existence of multiple zero profit prices. This is indicated by the green and blue curves in the figure.

Figure 2 represents on the left panel an example of the debt price correspondence for a given level of initial wealth W_0 , which is the result of repeating the previous analysis for a wide interval of government debt issuance levels. The curve on the right panel depicts the corresponding debt revenues. The set of zero profit prices is given by an inverted-S curve, and the function $Q(W_0, B_1)$ is a correspondence that maps from debt levels B_1 into a set of debt prices. If the government issues a low (high) amount of debt, it will be certain to get a high (low) price for it, because that is the only price consistent with the creditors' zero profit condition. If instead the government issues an amount of debt inside what I call the "multiplicity region", represented in the figure by all intermediate debt levels inside the $[\underline{B}_1, \overline{B}_1]$ interval, it may get either of the three zero profit prices consistent with it. It is important to note that both the bounds and the width of the multiplicity interval are a function of W_0 .

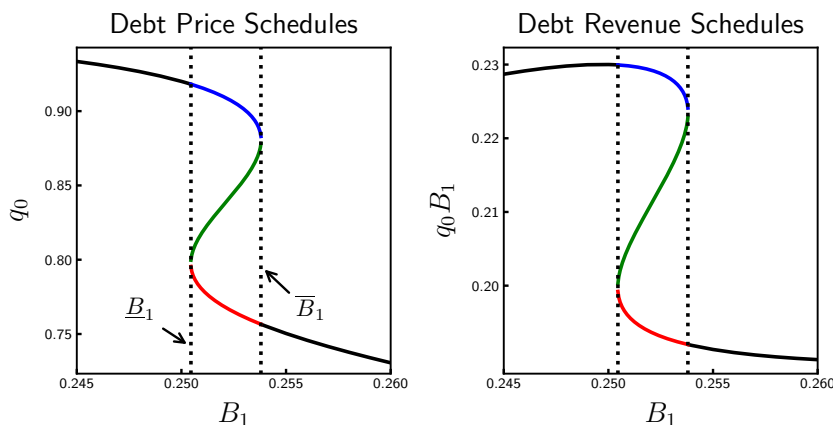


Figure 2: Debt price and debt revenues schedules.

3.3. Timing and Creditors Coordination

From now on, I split the debt price correspondence into single-valued schedules. Let us first note that, for all debt levels outside the multiplicity region, the correspondence is single-valued already. This will be the "common" part of any schedule, depicted in black in Figure 2. I define as the "good" schedule $Q^g(W_0, B_1)$ the function composed by the upper envelope of the curve in the multiplicity region (in blue), together with the common part. This curve will thus feature a discontinuous drop in the price of debt at \overline{B}_1 . Similarly, I define as the "bad" schedule $Q^b(W_0, B_1)$ the function composed by the lower envelope of the curve in the multiplicity region (in red), together with the common part. For this schedule, the discontinuity happens at \underline{B}_1 . Lastly, note that the green part of the curve inside the multiplicity region is unstable, in the sense that it is upward sloping. If the government were restricted to choose a point in that subset of the correspondence, it would always choose the largest possible debt level because that would fetch the highest price. For this reason I ignore such part of the debt price correspondence in the analysis.

This criterion to discipline coordination among lenders minimizes the number of discontinuities in each of the price schedules, and offers a clear ranking of schedules from the point of view of the government-borrower. In this example, I choose the two points of discontinuity that coincide with the boundaries of the multiplicity region.¹⁴

¹⁴Other pairs of price schedules obtained choosing any two discontinuity points inside the multiplicity region would also

Finally, I assume that at the beginning of period $t = 0$, the government knows which price schedule it will face *before* it issues new debt B_1 . The rationale behind this assumption is that, before auctioning off new debt, an issuer can observe conditions in the secondary market and understand *ex ante* at what price it may be able to issue a certain amount of debt. I thus interpret situations in which the government is facing the bad schedule as debt crises, or periods of market turbulence such as the European debt crisis, where sovereign borrowing becomes more expensive and investors are particularly concerned with default risk. In such times the government realises that, if it were to issue a level of debt inside the multiplicity region, it would face high interest rates and raise little funds because lenders would coordinate on the bad schedule.

3.4. Government Policy and Equilibria

So far, I have shown that the conditions at which the government is borrowing new debt may depend on self-fulfilling beliefs on the side of creditors. The existence of multiple outcomes of the issuance game, i.e. multiple price schedules, is however a necessary but not sufficient condition for the existence of multiple equilibria. To have the latter, it is also necessary that debt policy indeed depends on which price schedule the government is facing. That is, since the government moves first and chooses the optimal amount of debt to issue, equilibrium tax and debt policies will be a function of creditors' beliefs insofar as the borrowing motive is strong enough to push the government to consider debt levels inside the region that features multiple debt prices.

I now characterize government debt policy and households' investment policy as a function of the initial state W_0 and the debt price schedule $Q^i(W_0, B_1)$ for $i \in \{g, b\}$. At any interior point where $Q^i(W_0, B_1)$ is differentiable, the optimality condition for government debt is given by¹⁵

$$\frac{1}{C_0} = \frac{\beta}{Q^i + B_1 Q_B^i} \frac{1 - G(\hat{z}_1)}{C_1^R} \quad (8)$$

which I obtain after taking the first-order condition of government problem (6) with respect to B_1 , and combining it with equation (7). Equation (8) shows that the interest rate on debt is the inverse of the marginal revenue from borrowing one additional unit. Q_B^i represents the derivative of the price function with respect to debt issuance B_1 : as is standard in sovereign default models, it represents the negative price effect of issuing an additional unit of debt.

Figure 3 illustrates the equilibrium. It plots households' lifetime utility V_0 , debt policy, households' investment, debt prices, taxation and terminal debt/GDP, all as a function of initial wealth. Dotted black lines denote the first best, i.e. the full commitment solution; blue lines denote the good equilibrium; red lines denote the bad equilibrium. All curves are truncated at the initial wealth level where the equilibria stop existing because positive consumption would not be feasible anymore. I now describe the equilibrium more in detail.

It is possible to characterise equilibrium policy by households and the government by dividing the initial wealth state space into two regions. The first region corresponds to initial wealth levels larger than W_0^{FB} , the level of which is illustrated in the bottom left panel of the figure. In this region, government policy is not constrained by default risk and it coincides with the efficient allocation in the absence of the two main frictions in the model: limited commitment to repay on behalf of the government, and investment externalities on the household side. The price of debt is given by the risk-free price, and is not sensitive to marginal changes in the level of debt issuance (that is, $Q_B^i = 0$ for all i). The Euler equations for government debt issuance and households' investment respectively become

$$\frac{1}{C_0} = \beta R \frac{1}{C_1^R}; \quad \frac{1}{C_0} = \beta f'(K_1) \frac{1}{C_1^R}. \quad (9)$$

Combining the two conditions, we get that unconstrained government policy is characterised by a constant level of investment, and a debt policy which is linear and decreasing in wealth. Moreover, it is possible to

satisfy the above-mentioned properties. I pick the two boundaries to make my point more starkly. Other criteria with more than one discontinuity are less compelling, because they would make the price locally increasing in debt issuance.

¹⁵Henceforth, I omit the arguments of Q^i to lighten notation.

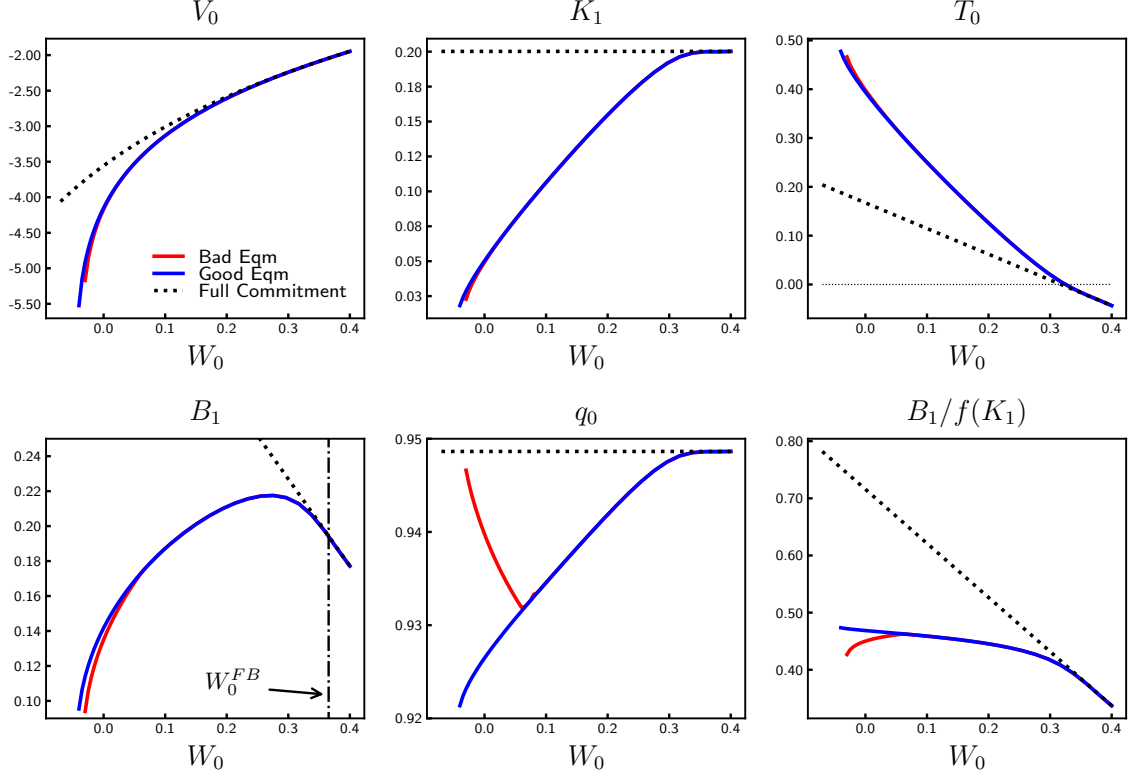


Figure 3: Policy functions and other equilibrium variables as a function of initial wealth. Notes: dotted black lines denote the first best (i.e. the full commitment solution); blue lines denote the good equilibrium under price schedule \mathcal{Q}^g ; red lines denote the bad equilibrium under price schedule \mathcal{Q}^b . Taxation T_0 in the top right panel is computed assuming $K_0 = K_1^{FB}$. All curves are truncated at the initial wealth level where the equilibria stop existing because positive consumption would not be feasible.

derive W_0^{FB} analytically: it is the lowest level of wealth for which the unconstrained policy is feasible. The following proposition formalises this point.

Proposition 1 (Unconstrained Policy). *Let z_1 be the largest value in the support of z_1 such that $\text{Prob}(z_1 \geq z_1) = 1$.¹⁶ Define*

$$W_0^{FB} := \frac{f(K_1^{FB})[1 - z_1(1 + \beta)]}{\beta R} + K_1^{FB}.$$

For all initial wealth levels $W_0 \geq W_0^{FB}$, the following statements hold:

- Investment is independent of initial wealth and is constant at the unconstrained level K_1^{FB} , defined by $f'(K_1^{FB}) = R$;
- Debt policy is linear, decreasing in wealth and is given by

$$B_1^{FB}(W_0) = \frac{f(K_1^{FB}) - \beta R(W_0 - K_1^{FB})}{1 + \beta}.$$

¹⁶I keep this argument general because it is valid regardless of the assumption on the distribution of z_1 . In the numerical example I use throughout the text, I assume that z_1 follows a truncated normal distribution, so that z_1 will coincide with the lower bound of the support of z_1 .

Proof. The algebra is straightforward: K_1^{FB} and B_1^{FB} are derived from the equations in (9), while W_0^{FB} is given by the solution to $\hat{z}_1(K_1^{FB}, B_1^{FB}(W_0^{FB})) = \underline{z}_1$. It is trivial to prove that the risk-free policy is the optimal one. ■

The second region of the initial wealth state space corresponds to wealth levels below W_0^{FB} , where the first best policy is not feasible because it would imply a future debt/GDP ratio at which default would occur with positive probability, given the government's inability to commit to repay debt. In this region, debt issuance is constrained by default risk, the price of debt is generally below the risk-free level, and households' investment is below the first-best. This happens because default risk raises the cost of borrowing, and the government partially substitutes debt issuance with taxation as the former source of funding becomes more expensive. Figure 3 illustrates this mechanism clearly: as wealth decreases below W_0^{FB} , taxation diverges from the first best and increases non-linearly, debt prices and investment drop below the risk-free level, and debt issuance initially increases at a slower pace, then decreases with wealth. Combining the optimality conditions for the government and the households yields

$$f'(K_1) \left[\frac{1 - G(\hat{z}_1)}{f(K_1) - B_1} + \frac{G(\hat{z}_1)}{f(K_1)} \right] = \frac{1}{Q^i + B_1 Q_B^i} \left[\frac{1 - G(\hat{z}_1)}{f(K_1) - B_1} \right]. \quad (10)$$

Equation (10) states that in equilibrium the government borrows up to the point where the marginal increase in interest rates (and related drop in debt revenues) due to a higher future debt stock is equal to the expected marginal (positive) effect on GDP of the corresponding tax cut. The following proposition formalises this point.

Proposition 2 (Risky Policy). *For all initial wealth levels below W_0^{FB} , equilibrium policy is such that:*

- *Capital investment is below the first-best level, $K_1 < K_1^{FB}$;*
- *When debt policy is such that Q^i is differentiable with respect to B_1 , debt is risky: $Q^i(W_0, B_1) < 1/R$ for $i \in \{b, g\}$.*

Proof. In Appendix B. ■

We can further divide this region of the initial wealth state space where policy is constrained into two sub-regions. For sufficiently high levels of initial wealth, debt issuance is constrained but the equilibrium remains unique: the borrowing motive of the government is not strong enough to push debt issuance inside the multiplicity region, and policy is unaffected by lenders' beliefs. This is where the blue and red curves in Figure 3 overlap. For low enough wealth levels instead, lenders' coordination failure affects equilibrium outcomes, and policy depends on whether the government is facing the bad schedule Q^b (red lines) or the good schedule Q^g (blue lines). Under the bad schedules, the government effectively faces a tighter borrowing constraint: issuing debt above \underline{B}_1 generates significantly lower revenues due to the sharp drop in debt prices. This can be seen clearly by looking at the right panel of Figure 2. In this case, government policy hits the de facto borrowing constraint \underline{B}_1 , and the second statement of Proposition 2 no longer applies because at that point the debt price function is not differentiable. For this reason, policy under the bad schedule is characterised by steeper price schedules, lower debt issuance, higher debt prices, lower debt revenues, higher $t = 0$ taxes, lower private investment and lower welfare.

In the bad equilibrium debt constraints become tighter as initial wealth drops, so that debt issuance decreases faster than investment and terminal debt/GDP actually decreases with initial wealth. The difference in welfare, tax and investment across the two equilibria does not appear stark from Figure 3: this is simply a result of the parameters chosen for the numerical example, especially the relatively low utility curvature implied by log utility, a choice aimed at making the algebra more transparent. It is worth noting that the separation of the initial wealth state space in two regions where policy is either constrained by default risk or unconstrained is a general result that is independent of the parametrisation of the model. On the other hand, it is not possible to prove formally whether, in the constrained region, there exist multiple equilibria. The numerical example I present here is chosen with the purpose of illustrating a case where equilibrium multiplicity indeed exists.

Interpretation. The model results have several features that relate to debt crises in general and the recent European debt crisis in particular.

First, the model offers a simple framework to analyse the debate over austerity policies that followed the European debt crisis. An important aspect of such debate concerns the question of whether running higher primary surpluses to reduce the debt stock indeed reduces debt/GDP, or is self-defeating because surpluses depress GDP more than they reduce debt. The optimality condition in (10) offers a way to look at this question through the lens of the model. In the model, borrowing one more unit of debt allows to increase debt revenues by an amount that depends on both the level and the sensitivity of the default risk premium present in bond prices. An increase in debt revenues allows to reduce taxation and increase households' consumption and investment. The marginal product of private investment can be directly interpreted as the multiplier on (a cut in) taxation. The model thus clearly highlights, on one hand, the role of default risk in the funding strategy of the government (i.e. in the choice between debt and taxes) and, on the other hand, the effect of fiscal consolidation on growth.

Second, the role of lenders' beliefs on equilibrium outcomes in the model sheds light on a crucial feature of the European debt crisis. I interpret the equilibrium under the bad schedule as a confidence crisis to which the government responds with austerity measures: lenders are pessimistic about fiscal policy and default prospects, and are only willing to lend to the government at high interest rates. Issuing large amounts of debt at these rates would be prohibitively costly (and raise very little revenues), so austerity becomes the optimal policy response because taxation is the cheapest source of funding. The shift in the government funding strategy from debt issuance to taxation depresses private consumption and investment, generating a vicious circle that results in a self-fulfilling debt crisis. The empirical regularities of procyclical fiscal policy and capital inflows, typically observed in emerging market economies, are consistent with these results: in a recession, default risk is high, debt issuance is costly, external borrowing and capital inflows are low, and fiscal policy is contractionary.

The model is reminiscent of the experience of some Southern European countries, such as Italy and Spain, during the European debt crisis. In the face of a confidence crisis and a spike in sovereign yields, these countries adopted a number of fiscal consolidation measures, such as tax hikes and spending cuts, aimed at increasing primary surpluses and reducing public debt. These measures were mainly motivated by the turmoil in sovereign debt markets, and arguably had two goals: to simply shift the government funding strategy from costlier debt issuance to primary surpluses, and to signal to financial markets that governments were serious about fiscal discipline and bringing public finances back to a sustainable path. In this paper, I focus on the first of these two motives, and I show that it has the potential to generate a vicious circle between bond prices and fiscal policy that triggers a debt crisis.¹⁷

The behaviour of equilibrium debt prices and revenues in the good and bad equilibria is one aspect of the model that deserves further discussion. While the bad debt price schedule offers weakly lower prices than the good schedule for any given level of debt, debt policy is such that the government endogenously restricts borrowing when it is facing the bad price schedule, so that equilibrium debt prices are actually higher in the bad equilibrium. This result stems from a number of assumptions that have the purpose of keeping the model tractable and the analysis transparent, the most important of which are the two period structure, the distribution of z_1 and the functional form of recovery upon default. There are a number of ways in which the result could be reversed, that would however require a substantial extension and complication of the model. It is however important to note that what matters for my results is that, in a confidence crisis, the government finds it optimal to substitute debt issuance with taxation. This allows to discuss austerity within the context of the European debt crisis, and is driven by a tighter constraint on the ability of the government to raise debt revenues. Whether debt revenues in the bad equilibrium are accompanied by higher or lower debt prices is, to conclude, a feature of the particular structure of the model and could be overturned in a less stylised exercise. In Section 5 I discuss more in detail how my assumptions are related to the results and to these extensions.

¹⁷It is unclear whether the second motive would have a similar or opposite effect. On one hand, it would provide a stronger incentive to do fiscal consolidation; on the other hand, it should have a positive impact on lenders' beliefs.

4. Policy Interventions

As highlighted throughout the exposition so far, the model results are driven by three main frictions. First, the government lack of commitment to repay debt at $t = 1$ constrains its future debt/GDP ratio. This friction introduces default risk premia in debt prices and affects the equilibrium for all wealth levels below W_0^{FB} , regardless of lenders' coordination issues. Second, the government's inability to commit to fiscal policy (i.e. taxation) before issuance at $t = 0$ makes tax policy dependent on what happens in sovereign debt markets. As I show, when initial wealth is low, lenders' coordination on pessimistic beliefs can be self-fulfilling and force the government to increase taxes and depress private investment. Third, households are atomistic and fail to internalise the effect that aggregate investment has on default risk and debt prices. This externality implies that private investment is inefficiently low in the presence of default risk.

I now consider a number of policy interventions that can help alleviate some of these frictions. First, I will consider the effects of the intervention of a single, large external lender, which for convenience I henceforth refer to as the IMF. This is closely related to the role that the IMF or the ESM can play during debt crises in emerging markets and Eurozone countries respectively. Second, I analyse domestic policies such as commitment to fiscal policy and private investment subsidies.

4.1. Non Defaultable Debt

Clearly, allowing for a technology that lets the government commit to repay debt would achieve the first best outcome for all initial wealth levels, and solve all frictions at once. This would amount to increasing default costs in the model enough that the temptation to default never arises.

One way this could be achieved is the IMF granting loans to the government that cannot be defaulted upon, because defaulting on the IMF is somehow much costlier than defaulting on private lenders. In this sense, facing a different lender could allow the government to reduce default frictions. Alternatively, the government could engineer an increase in (perceived) default costs by transferring the default choice to another institution (think of a central bank) that has a higher credibility (read higher perceived cost of default, or inflation, on private lenders) than the government. This is the logic through which central bank intervention can actively reduce the risk of belief-driven crises in sovereign debt markets.

In the case of unlimited IMF capacity, the outcome of the policy intervention would simply be given by the full commitment equilibrium represented by dotted lines in Figure 3. If instead the maximum amount of non-defaultable funds the IMF can lend is below the function $B_1^{FB}(W_0)$ for some wealth levels, the government would borrow a combination of IMF and market debt, with the latter remaining defaultable and being used to a minimum.

4.2. Pari-Passu Lending

I now consider the case where the IMF cannot extend non-defaultable loans, but participates in the standard bidding process together with private lenders, and is not senior to them. Crucially, I assume that the IMF is willing to lend to the government at the good price schedule, as long as that allows it to make zero profits in expectation. As in the previous case, the effects of this policy depend on the IMF capacity to lend.

Unlimited Capacity. If the IMF has unlimited capacity, then it can commit to buy debt at prices on the good schedule (black and blue lines of the example in Figure 2) for all levels of debt B_1 and wealth W_0 . This would avoid lenders' coordination failure and sustain the good equilibrium (blue curve in Figure 3). Since lenders are indifferent, the IMF would not need to actually lend any funds to the government, and its actions would work as a coordination device that selects the good equilibrium. It is worth noting that, in the case of unlimited capacity, pari-passu lending, senior lending or price floor policies would all be equivalent.

Limited Capacity. If the capacity to lend of the IMF is limited, then its commitment to buy a share of the government debt issuance at the good schedule price may or may not be sufficient to rule out coordination failure among private lenders.¹⁸ Let us consider this case: debt revenues are now given by $q_0^{imf} B_1^{imf} + q_0^{mkt} B_1^{mkt}$, where the *imf* superscripts are self-explanatory and the *mkt* superscripts denote the price paid and the debt level purchased by the market, i.e. private lenders. Given the IMF is not senior to private lenders, we can continue to express debt repayment obligations simply as $B_1 = B_1^{imf} + B_1^{mkt}$.

Figure 4 below is the analogue of Figure 1 and shows how the share of debt purchased by the IMF affects the behaviour of the debt price correspondence. The left panel shows the expected value of debt as a function of q_0^{mkt} , assuming that $q_0^{imf} = Q^g(W_0, B_1)$, that is, the IMF buys its share of debt at a price equal to the “good” zero profit price, represented by the highest round marker in the left panel.¹⁹ Each colour represents a different IMF intervention share. The right panel plots the corresponding private sector investment levels. The solid line is the equivalent of the “ B_1 medium” line of Figure 1. The three circles on the left panel highlight the zero profit prices for such level of debt and wealth.

By construction, the good zero profit price must still exist, as it is the price where all lenders (regardless of whether they are private or official) coordinate on the good price. This is represented by the highest circle in the left panel of the figure. The interesting part is what happens to the bad zero profit price. With IMF

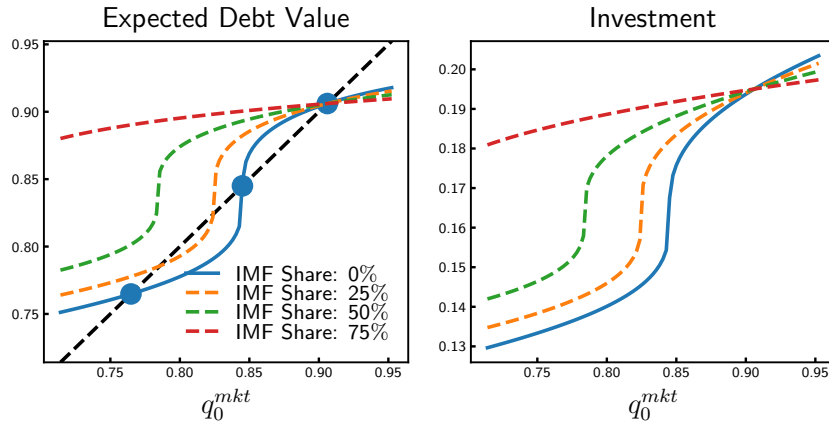


Figure 4: The effect of market debt prices on investment and debt value with pari-passu IMF interventions. Notes: the figure plots expected debt value and zero profit prices (left panel) and capital investment (right panel) as a function of q_0^{mkt} , for a given W_0 and conditional on q_0^{imf} being equal to the good zero profit price (i.e. the highest round marker), for different shares of debt purchased by the IMF. For this example, $B_1 = 0.2525$.

intervention, the pessimistic beliefs of private lenders have a limited impact on debt revenues, investment and in turn default probabilities. This implies that the value of debt is less sensitive to private lenders’ beliefs, which reduces the extent to which coordination failure is possible. This mechanism is stronger, the larger the share of debt purchased by the IMF. Essentially the IMF is offering the government a floor on debt revenues that prevents taxes and investment to respond too much when the market is not keen on buying government debt. In the limit where $B_1^{imf}/B_1 \rightarrow 1$, the market price obviously becomes irrelevant.

In the situations where IMF intervention would not rule out the bad price, the intervention itself is not implementable because it would imply an expected loss for the IMF if private lenders were to coordinate on pessimistic beliefs. Conditional on a IMF share, I thus assume that the IMF intervenes only at levels of issuance where its intervention is effective in ruling out coordination failure.

¹⁸ Assuming that the IMF buys a fixed share of total debt, rather than a certain level, may be a realistic assumption in light of the ECB capital key requirements, for example.

¹⁹ Note that it does not make a difference here whether the game between the IMF and private lenders is contemporaneous or has the IMF move first, or whether the IMF participates in the debt auction or extends direct loans.

The red lines in Figure 7 illustrate an example of equilibrium policy, debt prices and welfare in the pari-passu case, assuming lenders coordinate on the bad schedule when that is an equilibrium, and that the IMF buys up to 50% of issued debt. As the figure shows, IMF intervention offers a substantial improvement upon the baseline model, as it prevents private lenders' coordination failure for a larger set of initial wealth levels.

4.3. Senior Lending

I now consider the case where IMF loans are senior to private lenders. This is an intermediate (and perhaps more realistic) case between the non defaultable case (Subsection 4.1) and the pari-passu case (Subsection 4.2).

From the assumption on recovery upon default, we know that the total amount of resources that has to be distributed among creditors in a default is $\eta z_1 f(K_1)$, a fraction of the output loss. Figure 5 displays a graphical illustration of the payoffs to junior and senior lenders conditional on a default and as a function of the default cost z_1 . Let $z^{imf} := B_1^{imf} / [\eta f(K_1)]$ denote the lowest level of output loss upon default at which

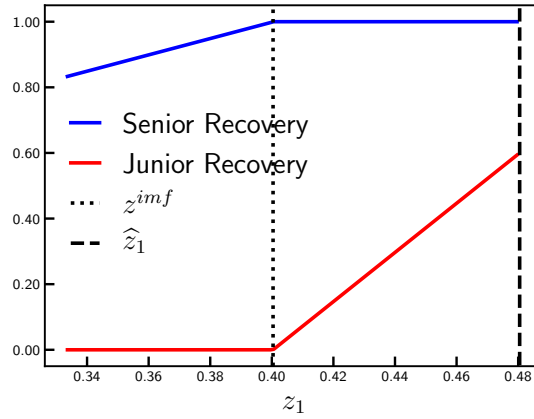


Figure 5: Junior and senior recovery upon default. Note: for this example, $K_1 = K_1^{FB}$ and the IMF debt share is 25%.

the IMF recovers all of its debt holdings. When a default comes with large output losses, i.e. $z_1 \geq z^{imf}$, then the IMF gets repaid in full and junior lenders receive $[\eta z_1 f(K_1) - B_1^{imf}] / B_1^{mkt}$, which is decreasing in z_1 . When a default comes with small output losses, i.e. $z_1 < z^{imf}$, then the IMF gets a partial repayment of $[\eta z_1 f(K_1)] / B_1^{imf}$ and junior lenders get nothing.²⁰

The junior and senior debt zero profit conditions are thus different and are given by

$$q_0^{imf} = \frac{1}{R} \left[1 - G(z^{imf}) + \int_{z^{imf}}^{z^{imf}} \frac{\eta z_1 f(K_1)}{B_1^{imf}} dG(z_1) \right]$$

$$q_0^{mkt} = \frac{1}{R} \left[1 - G(\hat{z}_1) + \int_{z^{imf}}^{\hat{z}_1} \frac{\eta z_1 f(K_1) - B_1^{imf}}{B_1^{mkt}} dG(z_1) \right]$$

where \hat{z}_1 and z^{imf} both depend on $B_1 = B_1^{imf} + B_1^{mkt}$ and K_1 . Debt revenues are given by $q_0^{imf} B_1^{imf} + q_0^{mkt} B_1^{mkt}$, and K_1 is given by the usual optimality condition for investment.

²⁰The assumption that total recovery upon default depends on output but not on debt is specifically intended to allow to discuss the role of creditors' seniority. An alternative assumption, that total recovery is a constant fraction of B_1 , would imply that recovery per bond is independent of investment, eliminating the role of lenders' beliefs.

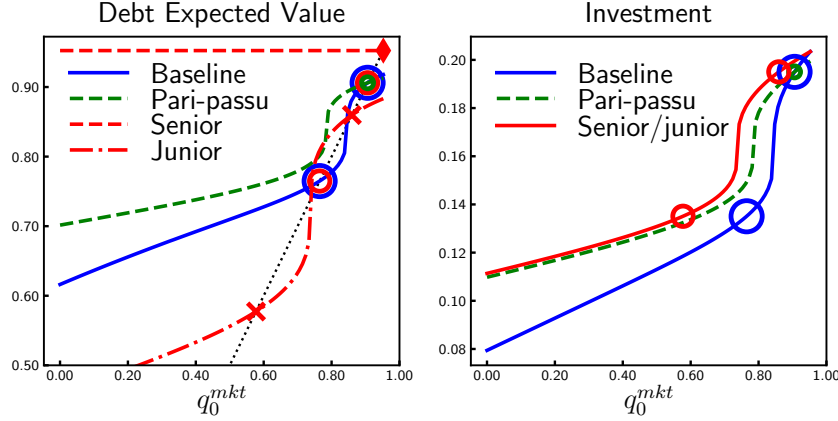


Figure 6: The effect of market debt prices on investment and debt value with senior IMF interventions. Notes: the figure plots expected debt value and zero profit prices (left panel) and capital investment (right panel) as a function of q_0^{mkt} . The blue line represents the case with no IMF intervention, as per Figure 1. The green line represents the case with IMF pari-passu intervention as per Figure 4. The red lines represent the senior and junior debt value (dashed and dash-dotted lines, left panel) and aggregate investment (solid line, right panel) as a function of q_0^{mkt} , assuming that $q_0^{imf} = 1/R$. In the left panel, red diamond and cross markers represent zero profit debt prices for senior and junior lenders. Round red markers represent the average debt price fetched by the government. All round markers on the right panel indicate the levels of investment corresponding to the different zero profit prices. For this example, the IMF debt share is 50%.

Figure 6 illustrates the difference between pari-passu and senior IMF interventions for given debt issuance and initial wealth levels. The left panel plots the expected value of debt in the different scenarios, with markers indicating zero profit prices, and the right panel plots the corresponding private sector investment. All variables are plotted as a function of q_0^{mkt} . For this example I fix the IMF share of government debt issuance to a value of 50%.

The blue line represents the baseline case with no IMF intervention ($B_1^{imf} = 0$), as per Figure 1. There are two stable zero profit prices indicated by blue circles on the left panel.²¹

The green line represents the case of IMF pari-passu intervention, conditional on q_0^{imf} being equal to the highest zero profit price (green round marker on the left panel), as per Figure 4. There exists one zero profit price, because in this example IMF intervention is effective in preventing private lenders' coordination failure.

The red lines represent the expected value of debt for senior and junior lenders (dashed and dash-dotted lines respectively, left panel) and aggregate investment (solid line, right panel) as a function of q_0^{mkt} , conditional on $q_0^{imf} = 1/R$ (that is, IMF lending is risk-free). In the left panel, red diamond and cross markers represent zero profit prices for senior and junior lenders respectively: there exists a unique (risk-free) senior price, but multiple stable junior prices. The red round markers represent the corresponding average debt price fetched by the government: there exist two, that depend on whether junior lenders coordinate on the good or bad zero profit price. The round markers on the right panel indicate the levels of investment corresponding to the different zero profit prices.

It can be proved that any solution to the pari-passu zero profit condition must also be the average of the solutions to the senior and junior zero profit conditions. The reverse is not necessarily true: there may exist prices for the senior debt case that do not exist in the pari-passu case. The lowest red circle on the left panel is an example: the senior tranche only admits one price (the risk-free price), while the junior tranche admits multiple prices. In both cases the corresponding average debt price coincides with the baseline case.

The intuition behind this result is the following: for a given q_0^{imf} , changes in q_0^{mkt} have a (small) marginal impact on investment. In the pari-passu case, this has a small effect on the expected debt value, because

²¹To lighten the figure I ignore unstable zero profit prices here.

the change in debt value is shared among *all* creditors. In the senior debt case, small changes in investment have tiny effects on the value of the senior tranche but may have large effects on the value of the junior tranche, especially when the thresholds \hat{z}_1, z^{imf} are such that junior lenders absorb all the change in the value of the debt, while the senior lender is insulated from it.

Senior debt is therefore more likely to give rise to coordination failure among private investors. This is reflected by the green lines in Figure 7, that illustrate the equilibrium in the senior debt case, assuming private lenders coordinate on the bad schedule when that is an equilibrium. The improvement upon the baseline case is moderate, and inferior to that caused by pari-passu lending, for the reasons explained in the previous paragraph.

There is however another important feature of the equilibrium under senior interventions: senior debt is always priced at the risk-free level, while pari-passu debt rarely is. This is visible in the bottom central panel of the figure, where green lines denote the average debt price (solid) and the senior and junior debt prices (dashed). In general, this aspect may be a relevant one to consider: when comparing pari-passu against senior lending, the lender's risk tolerance matters because there may exist a trade-off between the borrower's welfare and the lender's payoff variance.

4.4. Fiscal Commitment

A simple observation is that the government may be able to deter coordination failures by itself, if it were able to pick T_0 and B_1 at the same time and before the debt auction. This way, it would pin down K_1 , and in turn $q_0 = Q(K_1, B_1)$, uniquely. Essentially, the government would be selecting the debt price schedule, rather than take it as given. This could be interpreted as the conditionality clause often present in IMF (and ESM) bilateral lending agreements, if the technology that allows the government to commit to tax policy *ex ante* is somehow related to the IMF monitoring fiscal policy closely.

There are however two issues with this argument. First, commitment to fiscal policy is only useful as a coordination device if the IMF does not also participate in the government debt auction. If the IMF does not want to commit funds, then a role of policy monitoring or certification could be enough to avoid a market coordination failure. If instead the IMF were also stepping in by lending funds and selecting the equilibrium, then there would be no need for further coordination devices.

Second, this argument has a major problem related with government commitment on and off the equilibrium path. If the government commits to both T_0 and B_1 before the debt auction, then household investment is pinned down and the price of debt is uniquely determined. If T_0 and B_1 are such that the government budget constraint clears in $t = 0$, then we have an equilibrium. The problem with this approach is exactly that described in [Bassetto \(2005\)](#): the government is committing to actions, which implicitly assumes that at any off-equilibrium price q_0 the government budget constraint is simply violated, because there is no government policy left to adjust. Once we consider more seriously the role of the constraints that the government is forced to respect even away from equilibrium, it becomes necessary to specify a government strategy, rather than action, for how either taxes T_0 or new debt B_1 are set, which ensures the budget constraint holds on and off equilibrium. In this paper, I assume that taxes adjust. As I discuss in Section 5, papers in the spirit of [Calvo \(1988\)](#) make the alternative assumption that it is debt issuance that adjusts instead.

4.5. Investment Subsidies.

I now address both the second and third main frictions in the model: the government inability to commit to tax policy *ex ante*, and the externality in private sector investment.

Constrained Efficiency. To better understand these frictions, it is useful to consider as a benchmark the problem of a benevolent planner that can choose debt issuance and investment directly and contemporaneously, but remains subject to the inability to commit to debt repayment at $t = 1$. I refer to the solution to this problem as the constrained efficient allocation.²²

²²This is the framework used by the papers mentioned in footnote 8.

In this case, the price of defaultable debt $\mathcal{Q}(K_1, B_1)$ is a direct function of both debt and capital. The government can pin down debt prices uniquely and the feedback loop between debt prices, tax policy and investment is eliminated.²³ The optimality conditions for debt and capital are respectively given by

$$\begin{aligned}\frac{1}{C_0} &= \frac{\beta}{\mathcal{Q} + B_1 \mathcal{Q}_B} \frac{1 - G(\hat{z}_1)}{f(K_1) - B_1} \\ \frac{1}{C_0} &= \frac{\beta f'(K_1)}{1 - B_1 \mathcal{Q}_K} \left[\frac{1 - G(\hat{z}_1)}{f(K_1) - B_1} + \frac{G(\hat{z}_1)}{f(K_1)} \right]\end{aligned}\tag{11}$$

where \mathcal{Q}_K represents the derivative of the debt price function with respect to capital investment K_1 .

When government borrowing is not constrained by the risk of default, debt is risk-free and marginal changes in debt issuance or capital investment do not affect the price of debt: $\mathcal{Q}_K = \mathcal{Q}_B = 0$. In this case, which is only feasible for $W_0 \geq W_0^{FB}$, government policy coincides with the first best allocation described in Section 3.4.

For lower levels of wealth, default risk constrains debt choices. In the constrained efficient allocation, the government can choose capital investment directly, and it internalises the effect of such choice on debt prices. The term $B_1 \mathcal{Q}_K$ represents this additional marginal benefit of investment, which makes the optimality condition for investment differ from that of households in (7). This difference can also be interpreted with respect to the timing of moves in the $t = 0$ period. In the baseline model, lenders move first, and K_1 is the best response to the debt auction outcome. In the constrained efficient allocation, the government picks investment first, and q_0 is the best response of lenders to government debt and investment policy. As discussed in Section 5, the assumption that capital investment is contractible is important even when it is the government that chooses it: without such assumption, then investment would be inefficiently low even in the planner's problem.

Investment Subsidies. Let us now return our focus to the baseline model. In the presence of default risk, investment is thus inefficiently low because of an externality on the side of households. One way to correct such externality is for the government to introduce investment subsidies τ_0^k that (i) are contractible with lenders, and (ii) reduce the cost of household investment at $t = 0$ from one to $(1 - \tau_0^k)$. This changes the households' optimality condition for capital to

$$\frac{1}{C_0} = \frac{\beta f'(K_1)}{1 - \tau_0^k} \left[\frac{1 - G(\hat{z}_1)}{f(K_1) - B_1} + \frac{G(\hat{z}_1)}{f(K_1)} \right].\tag{12}$$

Adding an additional policy tool with these properties has two important effects. First, the government can commit to set τ_0^k so that $\tau_0^k = B_1 \mathcal{Q}_K$ and equations (12) and (11) coincide. In this way the subsidy implements the constrained efficient allocation by eliminating the externality and delivering a higher level of welfare. Second, having an additional policy tool restores equilibrium uniqueness. The government can use a combination of subsidies τ_0^k and lump-sum taxes T_0 to compensate the effect of (off-equilibrium) debt prices on investment. In other words, with an additional policy tool the government can commit to an off-equilibrium strategy that insulates investment and the expected value of debt from changes in debt prices, thus making the curves plotted in Figure 1 flat.

The dashed black lines in Figure 7 illustrate equilibrium policy, debt prices and welfare in the case of investment subsidies: the equilibrium is unique and welfare is higher than in the good equilibrium, because coordination failure is avoided and the households' investment externality is corrected. It is important to note that, if the government were not able to commit to the subsidy, then only lenders' coordination failure would be resolved: the investment externality would remain and the equilibrium would coincide with the good equilibrium of the baseline model.

5. Discussion

I now discuss in more detail some possible extensions to the framework proposed in this paper, as well as the most important assumptions behind the results of the model.

²³In this setting, consumption is free to adjust to off-equilibrium prices, so the issues raised in Subsection 4.4 do not apply.

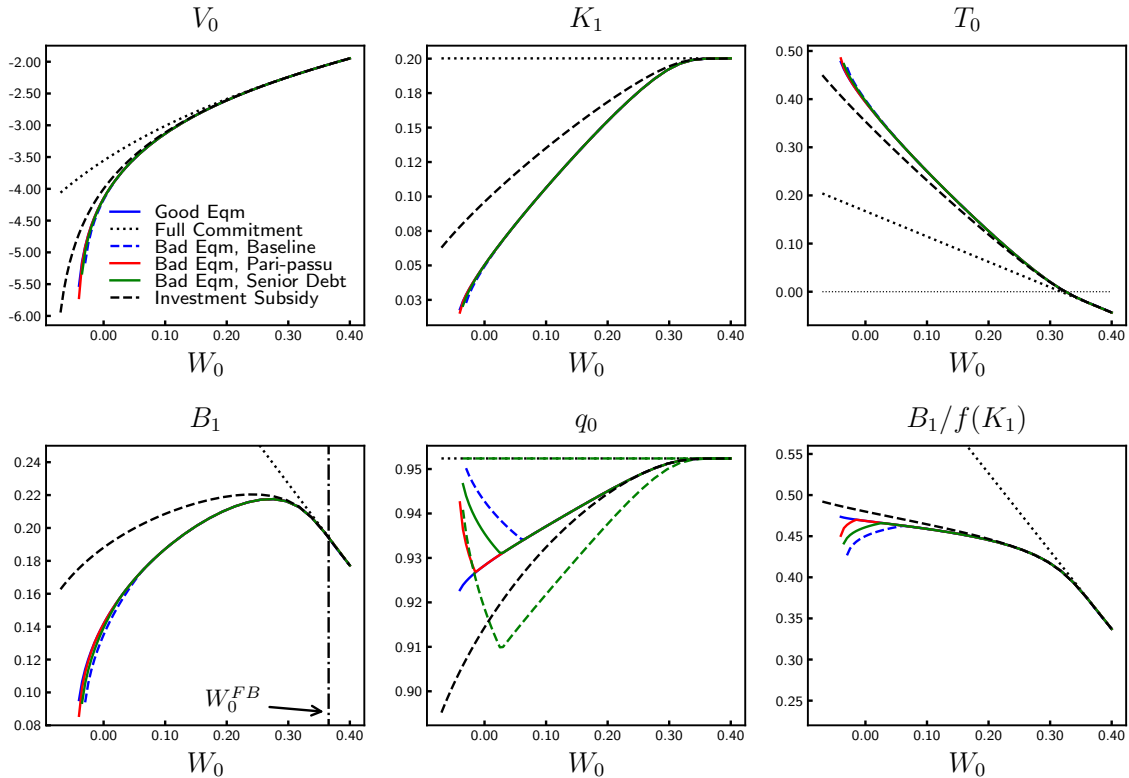


Figure 7: Policy functions and other equilibrium variables as a function of initial wealth. Notes: dotted lines denote the first best (i.e. the full commitment solution); dashed black lines denote the equilibrium under contractible investment subsidies (i.e. the constrained efficient allocation); solid blue lines denote the good equilibrium under price schedule Q^g ; dashed blue lines denote the bad equilibrium under price schedule Q^b ; solid red lines denote the bad equilibrium under IMF lending with a pari-passu status; solid green lines denote the bad equilibrium under IMF lending with a senior status. In the bottom central panel, green lines denote the average debt price (solid) and the senior and junior debt prices (dashed). Taxation T_0 in the top right panel is computed assuming $K_0 = K_1^{FB}$. All curves are truncated at the initial wealth level where the equilibria stop existing because positive consumption would not be feasible. For this example, the IMF debt share is 50%.

Extensions. Some of the economics in the paper are more general than the specific framework adopted here, and are complementary to mechanisms that have been explored by other, closely related strands of the literature. In the model, sovereign spreads have real effects on output through government fiscal policy, and the wealth effect of taxation on private investment. However, spreads may have adverse effects on economic activity in a number of other, different ways.

One possibility is that, in the presence of nominal rigidities, austerity policies hurt activity through aggregate demand channels. Consider a static, open economy model with tradable and non-tradable sectors and wage rigidities following, for example, [Na et al. \(2018\)](#) and [Bianchi and Mondragon \(2018\)](#). As in this paper, a debt crisis manifests itself with a sudden stop, i.e. a drop in capital inflows, and a corresponding increase in government taxation. In equilibrium this results in a trade surplus and lower tradable consumption. If the real wage cannot adjust, then there is a fall in demand for non-tradables, which creates involuntary unemployment. Nominal rigidities thus amplify the effects of a debt crisis by increasing the cost of debt repayment, and in turn the incentives to default. In a dynamic framework, this mechanism has the potential to generate self-fulfilling dynamics of a nature that is similar to, but different from, that analysed in this paper.

A second possibility is that the pass-through from sovereign spreads to real activity happens in a direct way, without the transmission of fiscal policy but through a disruption of financial intermediation, via the

so called sovereign-bank doom loop mechanism. When banks have substantial exposure to sovereign debt, a spike in spreads reduces the value of banks' net worth, hurting their ability to raise external funding and in turn lend to the private sector. This may increase firms' financing costs and constrain private lending, aggregate investment, and output. Works such as [Bocola \(2016\)](#), [Arellano et al. \(2017\)](#) and [Mallucci \(2015\)](#) among others have extended the financial frictions literature to analyse this mechanism in detail. [Corsetti et al. \(2013\)](#) show that, in the presence of nominal rigidities, sovereign risk not only harms economic activity via private funding costs, but also affects the size of fiscal multipliers.

Assumptions. To fully understand the forces at play in the paper, I now analyse more in detail the role that some assumptions have in driving the model results, and some of the properties of such results.

First, taxation is assumed to be lump-sum. The form of taxation is irrelevant in period $t = 0$, while it affects the optimality condition for investment in period $t = 1$. Assuming a proportional tax on production²⁴ creates an additional externality that reduces the marginal product of capital in repayment states, working against the wealth effect of taxation in period $t = 1$. This dampens the nonlinearity of the private sector investment response to the price of debt that I highlight in Subsection 3.1. In Appendix A I show that with log utility, the externality exactly offsets the wealth effect of taxes at $t = 1$, allowing to derive a closed form solution for $\mathcal{K}(W_0, q_0, B_1)$ which becomes a linear function of the household net income at $t = 0$. That allows to show in a transparent way that the crucial driver of multiplicity in the model are the distribution of default costs and the $t = 0$ wealth effect that debt prices have on investment via taxation. On the other hand, introducing a further externality makes the characterisation of government policy less transparent, and for this reason I choose to keep this model specification in the appendix.

Second, I assume that the structure of the government debt auction follows the Eaton-Gersovitz timing. That is, for a given debt price schedule known at the beginning of period $t = 0$, the government commits to a level of debt issuance first, and then lenders determine the price of debt.²⁵ As LW carefully point out, and as I make explicit in this paper, such timing structure implies that the government commits to adjust the primary surplus in order to satisfy the budget constraint, in case debt prices were different than expected. LW argue that in the short run it would be more plausible to assume that the margin that adjusts is debt policy rather than fiscal policy. If fiscal policy is then assumed exogenous, we get the framework of Calvo and subsequent work. On the contrary, in this paper I interpret the length of a period as long enough to allow a deterioration in government borrowing conditions to feed through to tax policy, and I focus precisely on the way in which fiscal policy adjusts to debt issuance outcomes.

Third, the fact that investment is carried out by a continuum of atomistic households naturally implies that tax policy is taken as given, and households do not internalise the effect of investment on the price of debt. If I were to consider a planner's problem where the government chooses investment directly, as [Detragiache \(1996\)](#) does, a further assumption would be needed, namely that the investment decision is not contractible with foreign lenders and is undertaken after the debt auction. This would ensure that investment responds to debt auction outcomes, which is the mechanism behind the existence of multiple equilibria. Again, I interpret the period length in my model as long enough that the variables affecting future output and default, such as private investment, are determined after the debt auction.

Fourth, in the bad equilibrium debt prices are higher than in the good equilibrium, as mentioned previously. I now elaborate on the reasons behind this result. First, it can be proved that the assumed functional form for recovery upon default implies that, when default probabilities approach one, debt revenues become constant and independent of B_1 . This implies that no debt level beyond \underline{B}_1 on the bad schedule allows to raise enough debt revenues to justify a large drop in prices. This is clearly highlighted by the right panel of Figure 2. The following extensions would alter this result and allow for the existence of high interest rates, high debt policy options that raise large debt revenues: a different form of recovery (e.g. assuming a per-bond recovery that is independent of B_1); a different timing structure (the Calvo structure allows for the costs of coordination failure, i.e. higher interest rates, to be partially avoided in case of default); longer debt maturities (with long term debt, a decrease in debt prices also reduces the value of the existing

²⁴This would be analogous to assuming taxes on income from capital or (inelastic) labour in a representative firm setting.

²⁵This timing structure is the same as that in the quantitative exercise of [Ayres et al. \(2019\)](#).

stock of debt). At the same time, however, one would also need that the government does choose these high debt, high rates policies. This could happen if the government is forced to so because it follows a fiscal rule, or if government utility has a high curvature that makes its borrowing motive strong when period $t = 0$ household consumption is low.

6. Conclusion

Default risk is a key determinant of sovereign borrowing costs, which have important implications for the joint dynamics of debt and fiscal policy, especially over the medium term and in countries with weak fundamentals and high debt stocks. This paper models in a simple and tractable way the circular relationship between government bond spreads, fiscal and debt policy, and economic activity.

I find that, under certain conditions, the expectations of sovereign debt investors may be self-fulfilling and, in a confidence crisis, induce a government to follow austerity policies that depress output and consumption. I believe that this can be an interpretation of the dynamics of southern European countries during the European debt crisis of 2010-12, and may be a useful framework for the analysis of scenarios where turmoil in sovereign debt markets affects debt and fiscal policy in meaningful ways.

I show that there are a number of policy interventions that can help to counter the frictions behind this vicious circle. *Pari-passu* and senior lending by an official external creditor can help reduce the extent of private lenders' coordination failure. Domestic policies that stimulate investment and are contractible with private lenders can help the government select the good equilibrium and correct the externality behind private sector under-investment.

This work focuses on a specific mechanism through which self-fulfilling debt crises might take place: sovereign credit risk affects economic activity via the response of fiscal policy. The next step would be to extend this analysis with a quantitative model and complement its key mechanism with other channels through which default risk has effects on debt policy and economic activity.

Appendix A Analytically Tractable Model

Here I present a two period model with the following simplifying assumptions. First, the government collects proportional taxes in $t = 1$. Second, I simplify lenders' zero profit condition by assuming $R = 1$ and that there is no recovery upon default (i.e. $\eta = 0$). This implies that the price of debt is equal to the probability of repayment. Third, I assume no discounting ($\beta = 1$) and a production function curvature of $\alpha = 0.5$, which helps to simplify some of the algebra. For now I continue to assume that the default cost is random as in the main text; later on I will consider the limiting case where the variance of the default cost distribution goes to zero, in order to draw sharper analytical conclusions.

With proportional taxation, the government budget constraint at $t = 1$ becomes

$$B_1(1 - \delta_1) = \tau_1 f(K_1)$$

where τ_1 is a proportional tax on production that equals $B_1/f(K_1)$ in case of repayment and zero in case of a default. The government default cutoff remains identical to that in equation (1).

What instead changes significantly is the household problem. The household budget constraint at repayment becomes

$$c_1^R = f(k_1)(1 - \tau_1).$$

There is now an additional externality: households take as given the proportional tax in repayment states, which reduces the marginal product of capital. The first-order condition for individual capital investment k_1 is now given by

$$\frac{1}{W_0 + q_0 B_1 - k_1} = \beta f'(k_1) \left[\frac{1 - G(\hat{z}_1)}{f(k_1)} + \frac{G(\hat{z}_1)}{f(k_1)} \right]. \quad (13)$$

This illustrates how the externality caused by the proportional tax assumption dampens the differential in the marginal product of capital between repayment and default states. Under the assumption of log utility,

the differential cancels out exactly. After imposing the equilibrium condition $k_1 = K_1$, we get that private sector aggregate investment is a constant fraction of after-tax household wealth in $t = 0$

$$\mathcal{K}(W_0, q_0, B_1) = \chi(W_0 + q_0 B_1) \quad (14)$$

where $\chi := \alpha\beta/(1 + \alpha\beta)$. This result holds true regardless of default probabilities, making the private sector investment response more tractable and highlighting the role of the default cost distribution in determining repayment probabilities. In fact, plugging equation (14) into the lenders' zero profit condition we get that any zero profit price must satisfy

$$q_0 = \text{Prob} \left(z_1 \geq \frac{B_1}{f(\chi(W_0 + q_0 B_1))} \right).$$

The effect of debt price variations on repayment probabilities is thus mainly due to the specifics of the

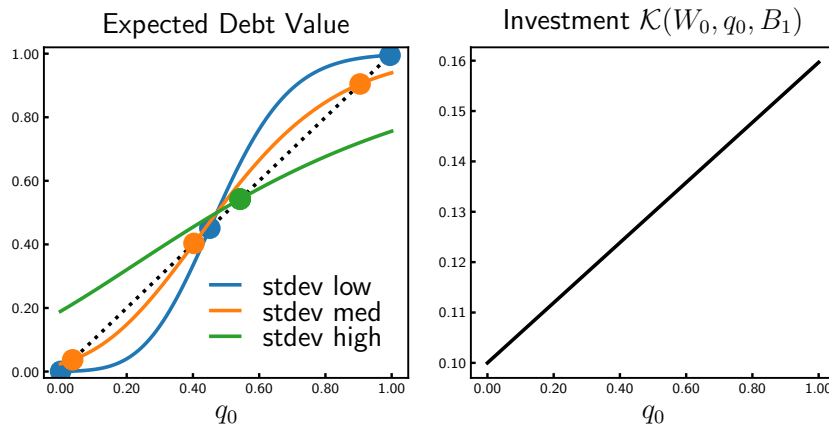


Figure 8: The effect of debt prices on investment and debt value. Notes: the figure plots expected debt value and zero profit prices (left panel) and capital investment (right panel) as a function of q_0 . The blue, orange and green lines respectively indicate parametrizations of the model where the standard deviation of z_1 is low, medium or high. This example uses the following values: $W_0 = 0.3, B_1 = 0.179$ and standard deviations of z_1 in $\{0.02, 0.0335, 0.075\}$.

distribution. Figure 8 plots repayment probabilities and investment responses for different standard deviations of the distribution G , keeping all other parameters unchanged. The right panel shows only one curve coloured in black, which represents the fact that the investment response is independent of the default cost variance. What affects the shape of the repayment probability curves in the left panel, and is a key driver of the existence of belief-driven multiple equilibria in this setting, is that the default cost distribution has an interior mode and a sufficiently low variance.

No Uncertainty. I now consider the limiting case where the distribution G is degenerate with all probability mass at a single point $\tilde{z} > 0$, which allows to go further with analytical derivations. The derivations of the main results can all be found in Appendix B. Let us denote with

$$\bar{K}_1(B_1) := \left(\frac{B_1}{\tilde{z}} \right)^{1/\alpha}$$

the threshold for private investment above (below) which the government finds it optimal to repay (default on) its debt B_1 .

First, I analyse whether there are debt levels for which there exist multiple zero profit prices. A sufficient condition for the existence of multiple zero profit prices is that, for a given (W_0, B_1) pair, the following two conditions are verified simultaneously:

$$\mathcal{K}(W_0, 0, B_1) < \bar{K}_1(B_1) \quad \wedge \quad \mathcal{K}(W_0, 1, B_1) \geq \bar{K}_1(B_1). \quad (15)$$

In words, this conditions means the following. If the government cannot borrow (i.e. faces a debt price of zero) because creditors anticipate a sure default, taxes in period $t = 0$ are high enough that private investment is below the repayment threshold; vice versa, if the government borrows at the risk-free rate (equal to one here) because creditors anticipate repayment, taxes are low enough that private investment is above the repayment threshold. The multiplicity region is thus given by the set of debt levels for which condition (15) holds.

We can characterise debt prices further using the closed form solution for \mathcal{K} and the assumption on the value of the production function curvature α . The lower bound of the multiplicity region, i.e. the debt threshold below which the government repays even when it gets a price of zero is

$$\underline{B}_1(W_0) := \tilde{z}\sqrt{\chi W_0},$$

which is increasing and concave in W_0 . The upper bound of the multiplicity region, i.e. the debt threshold above which the government defaults regardless of the price of debt is

$$\overline{B}_1(W_0) := \frac{\chi + \sqrt{\chi^2 + 4\chi W_0 \tilde{z}^{-2}}}{2\tilde{z}^{-2}}.$$

This defines explicitly the region of debt levels where there exist multiple equilibrium prices, $[\underline{B}_1(W_0), \overline{B}_1(W_0)]$. It can be proved that the width of such region is positive and decreasing in wealth W_0 .

Second, I characterise equilibrium policy. It is instructive to first write down the optimality condition for the government in its general form, abstracting from the simplifying assumptions made in this appendix:

$$\begin{aligned} & u'(C_0)(\mathcal{Q}_B^i B_1 + \mathcal{Q}^i) - \beta \frac{1 - G(\hat{z}_1)}{f(K_1) - B_1} + \\ & + \mathcal{K}_B \left[-u'(C_0) + \beta f'(K_1) \left(\frac{1 - G(\hat{z}_1)}{f(K_1) - B_1} + \frac{G(\hat{z}_1)}{f(K_1)} \right) \right] = 0. \end{aligned}$$

The last term, in square brackets, does not cancel out (as it instead does in the main text of the paper) because the additional externality in household investment introduced by proportional taxation implies that households invest less than the government would like them to. Let us now simplify the optimality condition under the special case of deterministic z_1 (so $\mathcal{Q}_B^i = 0$), and evaluate it at a debt level where repayment is certain (so $G(\hat{z}_1) = 0$, $\mathcal{Q}^i = 1$ and $\mathcal{K}_B = \chi$), which amounts to assuming that debt issuance is unconstrained. We get that any interior solution must satisfy the following optimality condition

$$\frac{f(K_1) - B_1}{C_0} = \frac{1 - \chi f'(K_1)}{1 - \chi}. \quad (16)$$

Combining equations (13) and (16) it is possible to show that in equilibrium $f'(K_1) > 1$, so capital is always below its first best level. Equation (16) is the analogue of equation (9), and implicitly defines unconstrained government debt policy $B_1^U(W_0)$ as function of the state W_0 .²⁶ It is possible to show that such policy is decreasing in W_0 , and therefore stops being feasible when wealth is low enough to prescribe a level of borrowing which is not compatible with the government's incentives to repay. It is however not possible to characterise such policy in closed form.

As in the main text, I define the “good” schedule as that under which the government can borrow risk-free up to $\overline{B}_1(W_0)$, and the “bad” schedule as the one that limits risk-free borrowing to $\underline{B}_1(W_0)$. We now have all of the pieces to fully characterise equilibrium policy.

Proposition 3. *Denote with $\underline{W}_0, \overline{W}_0$ the initial wealth levels such that $B_1^U(\underline{W}_0) = \overline{B}_1(\underline{W}_0)$ and $B_1^U(\overline{W}_0) = \underline{B}_1(\overline{W}_0)$. Equilibrium policy is such that*

²⁶I use a different notation from the main text, B_1^U instead of B_1^{FB} , because here the proportional tax externality implies that the equilibrium never coincides with the first best allocation.

1. Under the bad schedule, equilibrium policy is given by

$$\begin{cases} B_1^U(W_0) \text{ for } W_0 \geq \bar{W}_0 \\ \underline{B}_1(W_0) \text{ for } W_0 < \bar{W}_0. \end{cases}$$

2. Under the good schedule, equilibrium policy is given by

$$\begin{cases} B_1^U(W_0) \text{ for } W_0 \geq \underline{W}_0 \\ \bar{B}_1(W_0) \text{ for } W_0 < \underline{W}_0. \end{cases}$$

3. The wealth thresholds at which multiple equilibria arise are such that

$$0 < \underline{W}_0 < \bar{W}_0.$$

Proof. See Appendix B. ■

Equilibrium policy is illustrated in Figure 9. Its features are analogous to those derived in the main text, except that the absence of uncertainty here implies debt is always risk-free. When the borrowing limits are not binding, policy is not constrained by default risk and thus independent of lenders' beliefs. Unconstrained policy here is significantly different in nature from the first best policy, because of the externality introduced by proportional taxes. When instead the borrowing limits bind, debt policy is constrained, and this happens for a wider range of wealth levels under the bad schedule, when lenders fail to coordinate. Coordination failure here corresponds to a debt run where lenders refuse to lend, but is analogous in spirit to the results of the main text: the government is constrained in the amount of debt revenues it can raise, so is forced to increase taxation and depress private investment.

Appendix B Proofs

Proof of Proposition 2. By Proposition 1, the unconstrained policy is not feasible when wealth is below W_0^{FB} . I then prove that (i) when debt is risky, capital is below K_1^{FB} , and (ii) when capital is below K_1^{FB} , then debt is risky. Because of the criterion used to select debt price schedules, $\mathcal{Q}_B^i \leq 0$ by construction.

Proof of (i). By equation (5) it must be that $[1 - G(\hat{z}_1)]/(\mathcal{Q}^i + B_1 \mathcal{Q}_B^i) \geq R$. Then equation (8) implies that $(C_0)^{-1} \geq \beta R (C_1^R)^{-1}$. Subtracting $(C_0)^{-1} G(\hat{z}_1)$ from both sides of equation (7), we get

$$[1 - G(\hat{z}_1)] \left(\frac{1}{C_0} - \beta \frac{f'(K_1)}{C_1^R} \right) = G(\hat{z}_1) \left(\beta \frac{f'(K_1)}{f(K_1)} - \frac{1}{C_0} \right).$$

Because $1/C_0 \geq \beta R/C_1^R > \beta R/f(K_1)$, it must be that $f'(K_1) > R$ and in turn $K_1 < K_1^{FB}$.

Proof of (ii). If debt were risk-free, then $G(\hat{z}_1) = 0$ and by (9) we would have that $f'(K_1) = R$. This contradicts the assumption that $K_1 < K_1^{FB}$. ■

Proof of Proposition 3 and derivations of the analytical model in the appendix. I focus on positive initial levels of wealth, which allows to draw sharper analytical conclusions.

First, I derive the multiplicity bounds using the closed-form expression for \mathcal{K} from equation (14) and the conditions in (15). The lower bound $\underline{B}_1(W_0)$ is given by b such that $\mathcal{K}(W_0, 0, b) = \bar{K}_1(b)$, which is straightforward to derive. The upper bound $\bar{B}_1(W_0)$ is given by b such that $\mathcal{K}(W_0, 1, b) = \bar{K}_1(b)$. That yields a second order equation in b which has only one positive solution. More simple but tedious algebra shows that $\bar{B}_1(W_0) - \underline{B}_1(W_0)$ is strictly positive and decreasing in W_0 .

Second, I show that in equilibrium $f'(K_1) > 1$. Combining the first-order conditions for households' capital (13) and government debt (16) we get that $f'(K_1) = 1 + B_1/(W_0 + B_1)$, which implies that $1 < f'(K_1) < 2$ as long as both W_0 and B_1 are positive, which I assume.

Third, I show that unconstrained government debt policy is decreasing in W_0 . Debt policy is implicitly defined by $H := f(\mathcal{K}(W_0, 1, B_1)) - (1 - \chi)(W_0 + 2B_1) = 0$. Using the fact that $f'(K_1) < 2$, we get that $H_W < 0$, $H_B < 0$ and therefore that $dB_1^U(W_0)/dW_0 < 0$.

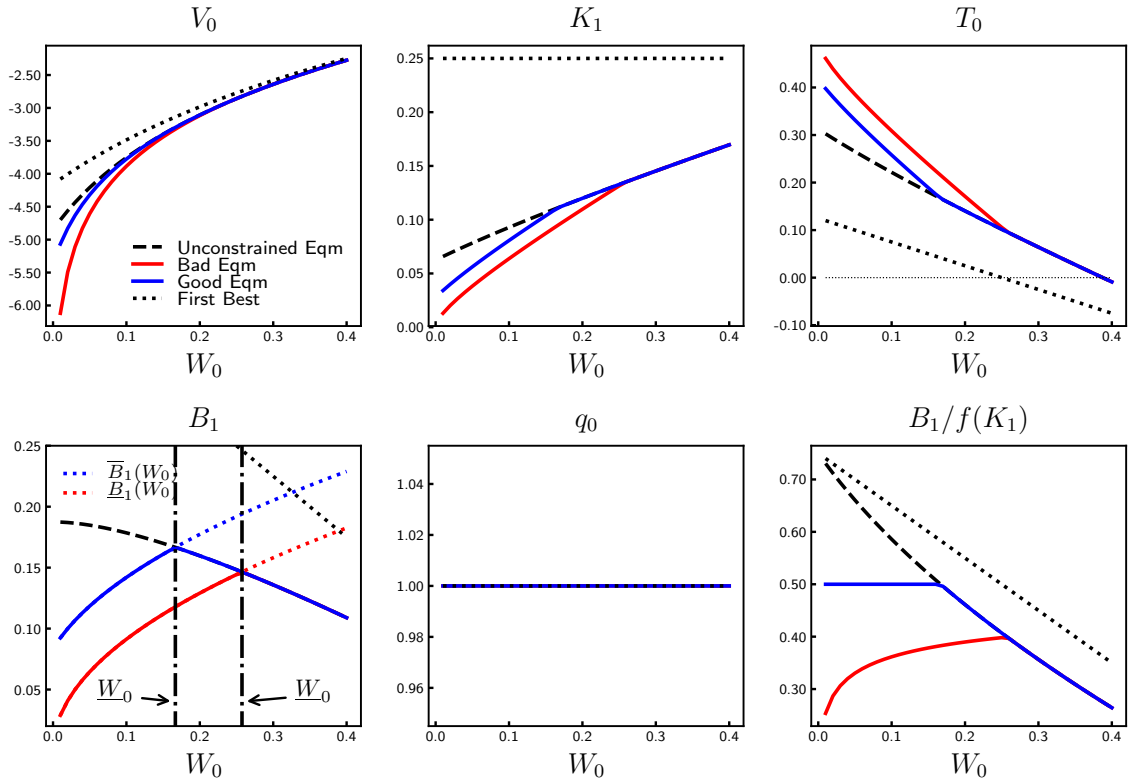


Figure 9: Policy functions and other equilibrium variables as a function of initial wealth. Notes: dotted black lines denote the first best (i.e. the solution of a planner's problem with full commitment); dashed black lines denote the unconstrained equilibrium; blue lines denote the good equilibrium under debt constraint $\bar{B}_1(W_0)$; red lines denote the bad equilibrium under debt constraint $\underline{B}_1(W_0)$. The bottom left panel also plots the debt constraint curves (the dotted coloured lines) and the wealth thresholds below which debt constraints bind. Taxation T_0 in the top right panel is computed assuming $K_0 = K_1^{FB}$.

Fourth, I show that $0 < \underline{W}_0 < \bar{W}_0$. The part $\underline{W}_0 < \bar{W}_0$ follows from the fact that unconstrained debt policy is decreasing in wealth and the multiplicity bounds are different and increasing in wealth. To prove that $\underline{W}_0 > 0$, I combine the implicit expression for $B_1^U(W_0)$ and the definition of \bar{B}_1 , and I obtain that their intersection must lie on the straight line given by $B_1 = W_0 \tilde{z}(1 - \chi) / [1 - 2\tilde{z}(1 - \chi)]$. This, combined with the fact that $B_1^U(0) > 0$, implies that $\underline{W}_0 > 0$. ■

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